Theoretical Performance of the Gradient-Based Tone Reservation PAPR Reduction Algorithm

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***Abstract*—In this paper, we provide a theoretical analysis on the performance of the famous Gradient-based Tone-Reservation PAPR reduction algorithm. To that end, we study the amplitude statistical distribution of the time domain OFDM signals to which PAPR reduction has been applied. Such distribution is then integrated into a generic EVM expression in order to provide a closed form expression of this latter, which is validated and analyzed through numerical simulations. Thanks to this work, we provide theoretical tools to analytically assess the performance of the Gradient method relatively to the optimal TR algorithm, and hence allow further analysis about the complexity-PAPR reduction efficiency trade-off of TR PAPR reduction algorithms, which could be useful for the optimization of the power efficiency of any multicarrier system.**

***Index Terms*—Analytical expression, EVM, OFDM, PAPR reduction, Probability distribution, Tone Reservation, Gradient algorithm.**

I. Introduction

Multi-carrier modulations and especially Orthogonal Frequency Division Multiplexing (OFDM) are key technologies in modern communication systems like Digital Video Broadcasting - Second Generation Terrestrial (DVB-T2) [1], Advanced Television Systems Committee 3.0 (ATSC3.0) [2], Long-Term Evolution (LTE) [3], as well as the recent releases concerning the fifth mobile generation (5G) [4]. The major issue related to such waveforms remains their high Peak to Average Power Ratio (PAPR). At the amplification stage indeed, the use of High Power Amplifiers (HPA) characterized by potentially strong non-linearities, brings about much more distortions to signals exhibiting high PAPR. The distortion level can then be mitigated by using the HPA at an operation point far from the saturation region, however dramatically degrading the HPA power efficiency. A fine tuning of the so-called Input power Back-Off (IBO) is thus seeked in practice to trade signal quality against power efficiency.

Two complementary ways can be followed to improve such distortion-power efficiency trade-off: either enlarging the linear zone of the HPA by means of predistortion techniques [5], or processing the baseband signal, in order to reduce its PAPR. In this paper, we are interested in this latter case. A large variety of PAPR reduction techniques exist in the literature, among which simple clipping approach [6] or the popular Tone Reservation (TR) concept [7]. Indeed, TR is a promising technique that has been proposed by standards like DVB-T2 and ATSC 3.0, because of its high efficiency in reducing PAPR without including distortions by itself and whatever the constellation order, and for its downward compatibility. TR consists in adding a kernel signal orthogonal to the transmitted signal in a way to reduce its PAPR without causing distorsions. Several algorithms can be used, in order to compute this kernel. The optimal one consists in solving a Quadratically Constrained Quadratic Problem (QCQP). Though this algorithm provides the optimal solution, it remains too complex to be implemented in real systems. For that reason, DVB-T2 and ATSC 3.0 rather consider a Gradient search based algorithm in practice. The Gradient solution provides a good complexity-PAPR reduction efficiency trade-off owing to the quite simple computation of the TR kernel at the transmitters.

In order to find the optimal system setting reaching the best distortion-power efficiency trade-off, it is needed to be able to predict the performance of the selected PAPR reduction algorithm. A large number of studies consider the performance of TR PAPR reduction and especially the development of the Gradient algorithm (see [8] and references therein). However, these studies are limited to empirical evaluations and the literature is lacking theoretical results. In this context, a recent study provided tools for the analytical computation of the Error Vector Magnitude (EVM) of OFDM signals after non-linear amplification and for any TR PAPR reduction algorithm [9]. EVM is an important figure of merit measuring the amount of in-band distortions of signals. The application guidelines of several standards like DVB-T2, ATSC3.0 and LTE have actually defined their requirements in terms of EVM. EVM is then well suited to assess the performance of PAPR reduction.

In this paper, we propose to derive the theoretical EVM of amplified OFDM signals after Gradient-based PAPR reduction as proposed by the DVB-T2 and ATSC3.0 standards. Following the methodology introduced in [9], we first provide a thorough analysis of the signal amplitude distribution after PAPR reduction, namely depending on the clipping factor which is the main setting parameter of the Gradient algorithm. The obtained statistical model, which is the heart of the material used for the theoretical EVM derivation is then used to derive the closed-form EVM expression for Gradient-based PAPR reduction. Hence, complementary to the expression

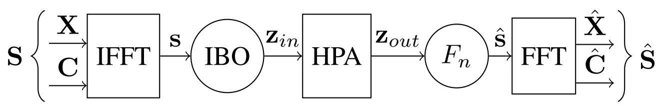

Fig. 1: System Model

given in [9], which has been derived for the QCQP algorithm, again not implementable in practice, we provide herein the EVM closed-form for the sub-optimal, but implementation-friendly, Gradient algorithm.

The rest of this paper is organised as follows. Section II presents the system model as well as the QCQP and Gradient algorithms. Section III is a summary of the analytical work done in [9] for the general case of TR technique applied for TR-QCQP. Section IV concerns the theoretical study of the Gradient algorithm. The amplitude distribution is analyzed and modeled. The EVM expression for the Gradient solution is thus evaluated. Section V presents numerical results. Section VI concludes this paper.

II. System Model

A. OFDM chain

Let us consider the OFDM transmission depicted in Fig.1. Without loss of generality, an OFDM system with $N = 8\ K$ subcarriers and 16-QAM modulation is considered using the DVB-T2 frame structure and its corresponding positions of Peak Reduction Tones (PRT). In the frequency domain, $\mathbf{X}$, $\mathbf{C}$, $\mathbf{S} \in \mathbb{C}^N$ present the original data signal, added signal by TR and resulting signal respectively. $\hat{\mathbf{X}}$, $\hat{\mathbf{C}}$, $\hat{\mathbf{S}}$ are their respective corresponding signals at the receiver. In the time domain, $\mathrm{s} \in \mathbb{C}^N$ of mean power P_s is the signal corresponding to $\mathbf{S}$, its received version is denoted by $\hat{\mathrm{s}}$. Before amplification, a power IBO is applied to s in a way to have a HPA input signal $\mathbf{z}_{in}$ of mean power P_{in} such that the IBO factor $F_{IBO} = \sqrt{P_{in}/P_s}$. The HPA output signal, $\mathbf{z}_{out}$, has a mean power P_{out} that is re-scaled to P_s by applying a normalization factor $F_n = \sqrt{P_s/P_{out}}$, in order to fairly compare constellation points of equal energy levels when evaluating the EVM. $\mathbf{C}$ is evaluated depending on the choice of the TR algorithm.

B. Tone Reservation PAPR Reduction

TR consists in adding the signal $\mathbf{C}$ to $\mathbf{X}$ in a way to reduce the PAPR of the resulting time domain signal s. Vector $\mathbf{C}$ is composed of R PRTs on the subset of sub-carriers S_l, vectors $\mathbf{X}$ and $\mathbf{C}$ lie into disjoint frequency sub-spaces such that $X_k = 0,\ \forall k\ \in S_l$ and $C_k = 0,\ \forall k \in \bar{S}_l$. This ensures that TR method does not include any distortion by itself. Let $\mathbf{Q}$ denote the inverse fast Fourier transform matrix, $\mathrm{c} = \mathbf{QC}$ is the TR kernel signal added to the time domain signal aiming at reducing its peaks. The evaluation of the kernel signal depends on the TR algorithm.

1) The QCQP problem: Basically, $\mathbf{C}$ is calculated based on the optimization problem introduced by Tellado [7] as follows:

$$\begin{aligned} &\min\ \tau \\ &\text{such that } \|\mathbf{x} + \mathbf{QC}\|_\infty^2 \leqslant \tau, \end{aligned} \tag{1}$$

where $\|.\|_\infty$ denotes the infinity norm. In standards, a power constraint is added to PRTs. Let $\Gamma = 10^{\frac{\Delta P_b}{10}}$ be the scale between the maximum power allowed to each PRT and the mean power of one data carrier P_{data}, where ΔP_b is the value of the power boost in dB, the following condition adds:

$$\|\mathbf{C}\|_\infty^2 \leqslant \Delta P_b + P_{data}. \tag{2}$$

The problem is thus a convex problem of type Quadratically Constrained Quadratic Problem (QCQP) [7]. The QCQP problem formulation is the optimal solution for TR and gives the upper-bound of PAPR reduction level using TR. However, QCQP is complex and introduces high latency due to its computation time. Several sub-optimal methods have been proposed in the literature [10]. The most famous is the gradient algorithm proposed by the DVB-T2 and ATSC3.0 standards that we describe in what follows.

2) The Gradient Algorithm: The concept of the Gradient algorithm described in DVB-T2 [2] is summarized in what follows. Each iteration of the Gradient algorithm aims at one peak reduction, in a way to ideally set all samples amplitudes under a predefined threshold V_{clip}. It consists on adding an impulse-like kernel to the time domain signal.

Note that as the number of PRTs is limited, the kernel is not a pure impulse and presents some leakage samples, which prevent from a perfect peak cancellation. In that sense, the Gradient-based algorithm is sub-optimal and, if it does not fail to, may need several tens of iterations to globally converge, depending on the threshold setting.

Based on such kernel, the peaks of the time domain signal can be reduced by shifting the kernel to the position of the highest peak. Let $y^{(i)}$ be the amplitude of the highest detected peak at the iteration i. The amplitude of the kernel is ideally equal to $|y^{(i)} - V_{clip}|$. However, this amplitude should respect the power constraint imposed on the PRTs in the frequency domain. Otherwise, the algorithm reduces the peak using the remaining amount of allowed power on the PRTs. For such case however, the sample amplitude is reduced, but is still above V_{clip}.

The algorithm continues this one peak cancellation by iteration until reaching one of the following breaking conditions. The first condition is reached when the amplitude of all the time domain samples remains under V_{clip}. The second one is obtained when the maximum allowed power ΔP_b is attained for at least one of the PRTs, whereas more power is required on this PRT to reduce the next peak. The third one is reaching a maximum allowed number of iterations I_{max} that maintains an acceptable computation time.

Throughout the paper, ΔP_b is set to 10 dB following the DVB-T2 and ATSC3.0 specifications, and I_{max} is set to 50. Please note that V_{clip} is referenced to a signal power $P_x = 1$.

C. HPA Model

A memoryless amplitude to amplitude characteristic of the HPA is considered by the following polynomial model:

$$\mathrm{H}_{\mathrm{HPA}}(r_{in}) = \sum_{l=0}^{L-1} b_{2l+1} {r_{in}}^{2l+1}, \tag{3}$$

where r_{in} is the HPA input amplitude, L the order of the HPA polynomial model and b_{2l+1} its odd indexed coefficients. The IBO $\rho_{_{IBO}}$ applied at the HPA is defined as the ratio between the HPA 1 dB compression point P_{1dB} and P_{in}.

D. EVM Definition

EVM is a metric that measures the in-band distortion level of the signal. It measures the amount of deviation of the received constellation points $\hat{\mathbf{S}}$ relative to the original frequency domain signal $\mathbf{S}$. Letting $\mathbb{E}(.)$ be the expectation expression, EVM can then be expressed by:

$$EVM = \sqrt{\frac{\mathbb{E}(||\hat{\mathbf{s}} - \mathbf{s}||^2)}{\mathbb{E}(||\mathbf{s}||^2)}}. \quad (4)$$

III. Generic EVM expression for TR PAPR reduction and its application to QCQP algorithm

In this section, we summarize the main results provided in [9] to establish the EVM expression of OFDM signals after TR PAPR reduction and non-linear power amplification. Its utilization with the TR-QCQP algorithm is then exposed as an application example to better understand how to further exploit the generic expression for Gradient-based algorithms in section IV.

A. EVM expression based on statistical signal description

As demonstrated in [9], a generic EVM expression derived from (4) can be obtained as a function of the time domain statistical amplitude distribution of the OFDM signal after TR PAPR reduction and non-linear amplification depending of the HPA model given by (3). It is expressed as:

$$EVM = \sqrt{2}\,(1-\eta)^{1/2} \quad (5)$$

$$\text{with,} \quad \eta = \frac{\sum_{l=0}^{L-1} u_l \; \rho_{IBO}^{l+\frac{1}{2}} \; \mathrm{m_s}\left(2(l+1)\right)}{\sum_{l,l'=0}^{L-1} v_{l,l'} \; \rho_{IBO}^{l+l'+1} \; \mathrm{m_s}\left(2(l+l'+1)\right)} \ ,$$

where $u_l = \frac{P_{1dB}^{l+1/2}}{P_s^{l+1}}\mathrm{Re}(b_{2l+1})$ and $v_{l,l'} = \frac{P_{1dB}^{l+l'+1}}{P_s^{l+l'+1}} b_{2l+1} b^*_{2l'+1}$. $\mathrm{m_s}(n)$ is the n^{th} ordered raw moment of the distribution $\mathrm{f_s}(r)$, time domain statistical distribution of the amplitude of the OFDM signal $r = |s|$ after applying TR, that is:

$$\mathrm{m_s}(n) = \int_0^\infty r^n \mathrm{f_s}(r)\mathrm{d}r \quad (6)$$

This expression presents a general analytical framework for the TR technique relying on the evaluation of the moments of the time domain amplitude distribution $\mathrm{f_s}(r)$.

B. Application to TR-QCQP

As studied in [9], it can be shown that the time domain distribution the OFDM signals after TR-QCQP processing is composed of two modes. The first one is the Rayleigh distribution of the original OFDM signal truncated at r_{step}. The second one follows a Generalised Extreme Value (GEV) distribution. Hence the signal amplitude distribution writes:

$$\mathrm{f_s}(r) = \frac{1-p}{q}\, \mathrm{f}_{\mathrm{Ray}}^{P_r}(r)\,(1-\mathrm{u}(r-r_{step})) + p\, \mathrm{f}_{\mathrm{GEV}}^{\mu_2,\sigma_2,k_2}(r) \quad (7)$$

where p is a scaling factor between the first and the second mode, and q is a normalization factor, P_r, μ_2, σ_2, k_2 are the parameters of the Rayleigh and GEV distributions. From such distribution model, the moments of the time domain distribution for TR-QCQP can be derived as follows:

$$\mathrm{m_s}(n) = \frac{1-p}{q}\; P_r^n \;\gamma\left(n+1,\Lambda\right) + p \sum_{p_1+p_2+p_3=2n} w_{2n}\; \Gamma\left(-k_2 p_1 + 1\right)$$

with $w_{2n} = \binom{2n}{p_1,p_2,p_3}(-1)^{p_2}\left(\frac{\sigma_2}{k_2}\right)^{p_1+p_2} \mu_2^{p_3}$ and $\Lambda = \frac{r_{step}^2}{P_r}$, γ and Γ present respectively the lower incomplete and the complete gamma functions. Thus, the EVM for TR-QCQP is obtained by substituting the moment in (5) by the above expression. This result was validated in [9] by proper EVM simulations that show the relevance of these analytical derivations.

From this example, it is understood that any EVM expression can be obtained provided that the raw moments of the signal amplitude distribution are available, as investigated in the Gradient-based case in the sequel.

IV. Study of the Gradient Method

This section deals with the theoretical performance derivation of the Gradient-based PAPR reduction, based on the exploitation of (5) The time domain amplitude distribution is first studied and analyzed for different parameters of the system. Then, a model of such distribution is investigated for a certain set of parameters. This model is finally integrated as in (6), in order to provide the analytical EVM expression.

A. Amplitude Distribution Analysis

Let us first analyze the empirical distribution for the time domain OFDM signal after applying the Gradient method. Fig. 2 shows such distribution for different values of V_{clip} and using the 8 K DVB-T2 frame structure. From this figure, one can remark that the algorithm begins to take effect on modifying the distribution starting from $V_{clip} = 2$, whereas before this value the output signal follows the original Rayleigh distribution, which characterizes OFDM signals. It is understood that the Gradient method translates into reducing the amount of samples above V_{clip} and somehow forcing them at V_{clip} in a way to reduce the peak values of the signal. This seems to be effective only for a sufficiently high value of V_{clip}.

To go further in the analysis, Table I shows statistics on the breaking conditions of the algorithm for different values of V_{clip} on a total of $T = 10000$ OFDM symbols. N_V, N_P and N_I are the number of symbols, where the algorithm exits respectively because (i) all the samples are under the limit of V_{clip}, (ii) the maximum PRT power is reached, or (iii) because the maximum number of iterations is reached. I_V and I_P are the mean number of iterations before breaking due to (i) or (ii) respectively. We can hence note that for high values of V_{clip} like 2.5 and 2.3, the algorithm succeeds in limiting the amplitude of all the samples under V_{clip} for a high percentage of symbols while respecting the power constraint. This reflects the feasibility of the algorithm for high V_{clip}

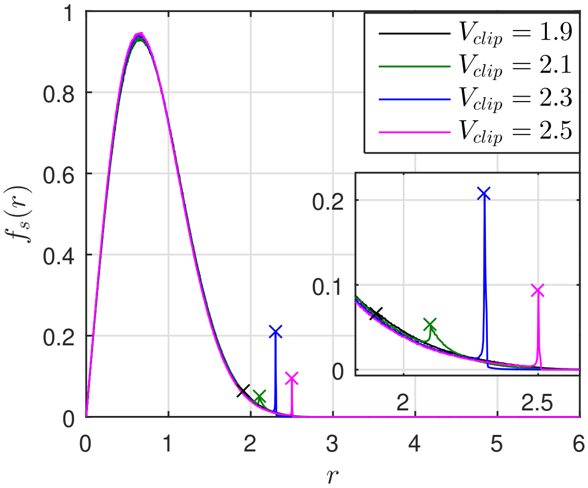

Fig. 2: Time domain amplitude distribution of the OFDM signal using the Gradient algorithm for different V_{clip}

TABLE I: Statistics on the breaking conditions of the Gradient method for different values of V_{clip} with $I_{max} = 50$ iterations

V_{clip}	N_I	N_V	I_V	N_P	I_P
1.7	0	0	–	10000	5.15
1.9	0	0	–	10000	6.48
2.1	666	25	44.76	9309	13.76
2.3	70	9014	21.99	916	16.15
2.5	0	9993	5.88	7	9.85

with $\Delta P_b = 10\ dB$. However, for $V_{clip} = 2.1$, the algorithm succeeds in his task for only 25 symbols, whereas for the rest of the symbols, the algorithm passes the power limit or the maximum number of iterations before succeeding to set all the samples under the threshold. This means that the algorithm should not be processed at low values of V_{clip}. It actually becomes inefficient in that case, since the assumption of a perfect Dirac-like kernel, onto which the algorithm is based, is not strictly valid. In fact, this makes the algorithm not correctly converge when V_{clip} is too low. In addition, for low V_{clip}, the algorithm rapidly passes the power constraint due to the large amplitude difference between V_{clip} and the highest peaks in the signal. This eventually explains why the output signal distribution is not modified in Fig. 2 when $V_{clip} < 2$.

B. Amplitude Distribution Modeling

From the previous section, we understand that the Gradient algorithm should be used for sufficiently high values of V_{clip}, in order to be effective. With $V_{clip} > 2.2$, one can note from Fig. 2 that the distribution is composed of two main modes. The first is the Rayleigh distribution of the original OFDM signal, which is truncated at V_{clip}, corresponding to the samples under V_{clip} resulting after applying the algorithm. The second reflects the number of samples around and at V_{clip} resulting from the modification of samples due to the algorithm.

We thus propose to model the distribution as a superposition of two functions. The first is the Rayleigh distribution, truncated at V_{clip}. The second is modeled as an impulse located at V_{clip}, which corresponds to an ideal behavior of the algorithm, as if it were able to perfectly force the samples above the threshold to take an amplitude of V_{clip}. Thus, the distribution can be expressed as:

$$\mathrm{f_s}(r) = \frac{(1-p')}{q'}\, \mathrm{f}_{\mathrm{Ray}}^{\mathrm{P'_r}}(r)(1-\mathrm{u}(r-V_{clip}))+p'\,\delta(r-V_{clip}) \quad (8)$$

where p' is the ratio of samples at V_{clip} and the total number of samples, $\mathrm{f}_{\mathrm{Ray}}^{\mathrm{P'_r}}(r)$ is the Rayleigh distribution of power parameter P'_r, $\mathrm{u}(r)$ is the step function and $\delta(r)$ is the Dirac delta function. As can be seen in Fig. 2, this approximation can not be valid before a limit of $V_{clip} = 2.1$, where the algorithm begins its reduction effect. The validity of this approximation will be later investigated when studying the EVM results in the following.

C. Analytical EVM Expression

From the above modeling of the distribution, the n^{th} moment of $\mathrm{f_s^{Grad}}(r)$ denoted by $\mathrm{m_s^{Grad}}(n)$ can be developed as:

$$\mathrm{m_s^{Grad}}(n) = \frac{(1-p')}{q'}\ \mathrm{m_{s_1}}(n) + p'\ \mathrm{m_{s_2}}(n)\ ,\ \text{with:} \quad (9)$$

$$\mathrm{m_{s_1}}(n) = \int_0^{V_{clip}} r^n \mathrm{f}_{\mathrm{Ray}}^{\mathrm{P'_r}}(r)\mathrm{d}r\ ; \quad (10)$$

$$\mathrm{m_{s_2}}(n) = \int_0^{\infty} r^n \delta(r - V_{clip})\mathrm{d}r\ . \quad (11)$$

From the integral properties of the Dirac delta function, $\mathrm{m_{s_2}}(n)$ is simple to compute. For $\mathrm{m_{s_1}}(n)$, using the definition of the Rayleigh distribution, it can be evaluated through:

$$\mathrm{m_{s_1}}(n) = \int_0^{V_{clip}} r^n \frac{2r}{P_x} e^{-\frac{r^2}{P_x}} \mathrm{d}r \quad (12)$$

From this level, using the integral properties in [11] for the exponential function and for the Dirac delta, the integration of $\mathrm{m_s}(n)$ for the Gradient method is straightforward. Thus, the EVM for the Gradient method can be expressed as in (5) with $\mathrm{m_s}(n)$ expressed as:

$$\mathrm{m_s^{Grad}}(n) = \frac{(1-p')}{q'} P_r'^{\frac{n}{2}} \gamma \left(\frac{n}{2}+1, \frac{V_{clip}^2}{P'_r}\right) + p'\ V_{clip}^n\ . \quad (13)$$

Thus, the closed form EVM expression of (5) for the gradient TR algorithm can be easily evaluated as a function of V_{clip}, which is the key parameter of the algorithm.

V. Numerical Results

In this section, numerical EVM results are analyzed. First, the validity of the proposed theoretical EVM expression is discussed by comparing simulation and analytical results. Then, an analysis of the performance of the Gradient algorithm is led, in order to investigate the optimal parameters of the Gradient algorithm in the DVB-T2 standard. Finally, the Gradient algorithm is compared to the TR-QCQP methods.

A. Validity of the EVM expression

Fig. 3 presents theoretical and simulated EVM in function of the IBO, with a power constraint for TR PAPR reduction equal to $\Delta P_b = 10\ dB$, $I_{max} = 50$ and different values of V_{clip}. As can be seen, the analytical results show a good match with the simulated EVM at high V_{clip} values. More precisely, the difference between the theoretical and simulated EVM is less than 0.2% for V_{clip} above or equal to 2.3. This validates the analytical EVM expression and the distribution model for high V_{clip} values. However, higher differences are noticed as V_{clip} values are lower, where the Gradient algorithm fails in efficiently forcing the samples to take the amplitude of the threshold. In that case, the approximation in the distribution model becomes visible. We then conclude that the approximation is valid for $V_{clip} \geqslant 2.3$.

B. Study of the DVB-T2 Gradient Algorithm Parameters

As mentioned above, the DVB-T2 standard specifies a power constraint for TR PAPR reduction to $\Delta P_b = 10\ dB$. In a way to guarantee an implementable algorithm on base stations with acceptable delays, the number of iterations should be under I_{max} that we limit to 50 iterations. According to the algorithm analysis of the breaking conditions in Table I, we hereby analyze the performance of the DVB-T2 Gradient algorithm in terms of EVM. Fig. 4 shows EVM values in function of V_{clip} for different values of the IBO using the above described DVB-T2 parameter setting using the DVB-T2 PRT positions for 8K subcarriers and $I_{max} = 50$.

As can be seen, an optimal performance of the algorithm is achieved for $V_{clip} = 2.1$. This can be explained by the fact that at greater values of V_{clip}, a small amount of samples is reduced due to the limited number of samples above V_{clip}. This can be seen in Table I, where N_V is near to the total number of symbols for $V_{clip} = 2.3$ and 2.5. However, for V_{clip} lower than 2.1, the power constraint is rapidly reached due to the high difference between V_{clip} and the highest detected peaks.

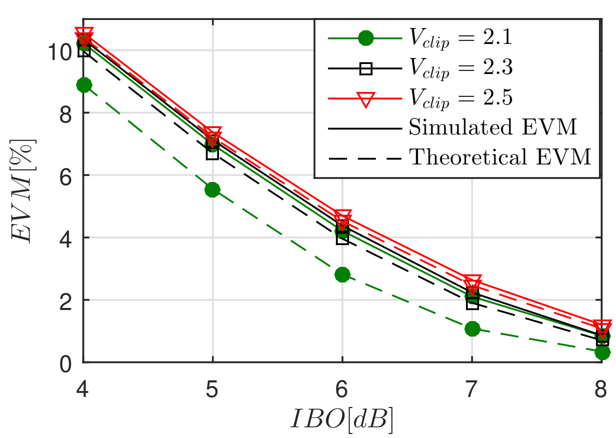

Fig. 3: Simulated and theoretical EVM vs IBO for different values of V_{clip}

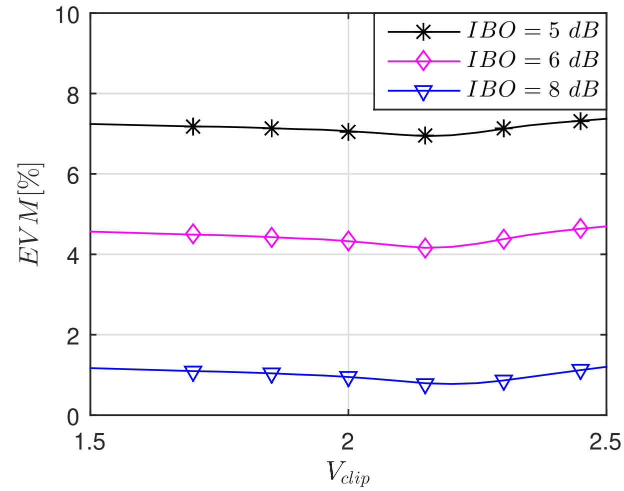

Fig. 4: EVM in function of V_{clip} for different IBO using Gradient algorithm with DVB-T2 parameters

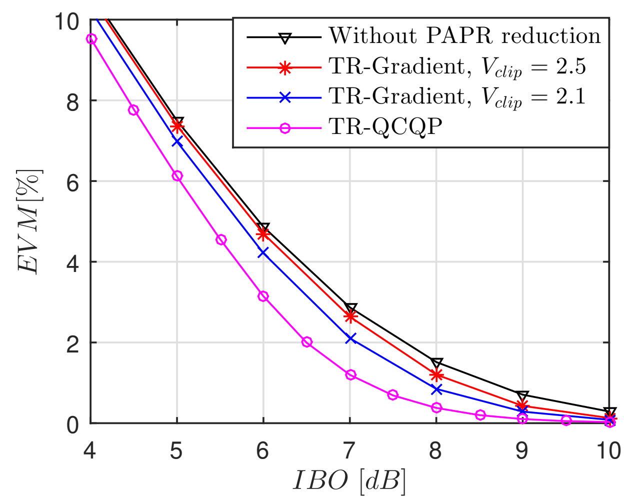

Fig. 5: Comparison of the EVM values using different PAPR reduction schemes

Indeed, $V_{clip} = 2.1$ provides the best compromise between the use of the allowed PRT power and the efficient peak reduction.

C. Comparison to TR-QCQP

Fig. 5 shows the EVM values using the Gradient, TR-QCQP techniques as well as the case without PAPR reduction. The DVB-T2 frame structure is considered with $\Delta P_b = 10\ dB$. As can be seen, the optimal performance is provided by QCQP algorithm that by turns costs complex implementation and computation time. However, the gradient method while using the $V_{clip} = 2.1$ provides an efficient EVM reduction relative to the case without PAPR reduction, whereas it is much simpler than QCQP. It thus visualises the compromise between PAPR reduction efficiency and the implementation complexity between QCQP and the gradient techniques.

VI. Conclusion

In this paper, an analytical study of the Gradient TR PAPR reduction algorithm is provided, following a generic framework. This framework is based on a statistical analysis of the signal after PAPR reduction. A theoretical approximated distribution of the samples amplitude of the signal is proposed as a bi-modal distribution. This model allows for rapidly deriving the theoretical EVM of the signal at the output of the PA and predict the algorithm performance. A good convergence between the theoretical and simulated EVM values for V_{clip} equal or higher than 2.3 is observed, which validates the used distribution model and the analytical approach for those V_{clip} values. Furthermore, the optimal parameters of the Gradient TR PAPR reduction have been determined based on simulation results taking into account realistic conditions of the DVB-T2 and ATSC3.0 standards. . Finally, a comparison to optimal TR algorithm is lead in order to assess the performance of the gradient method in reducing EVM, compared to such algorithms.

This work eventually provides theoretical results that can be useful for further comparisons and analysis between the Gradient-based and other PAPR reduction algorithms in the perspective of the usage of such schemes in future systems.

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