Diversity-Multiplexing Tradeoff for Indoor Visible Light Communication

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***Abstract*—In the past few years, visible light communication (VLC) has been a surge of interest in the research community. Different studies on VLC based communication systems were part of the research in the recent few years. However, there is no study on the development of the diversity-multiplexing tradeoff (DMT) metric to compare and design new multiple-input multiple-output (MIMO) VLC schemes. This paper introduces the concept of DMT for indoor VLC-based communication systems that utilize intensity modulation and direct detection technique for which channel coefficients are real and positive. DMT is characterized as a vital performance metric for comparing different MIMO techniques. For simplifying the presentation, we start with a single-input single-output VLC system and broaden this analysis to MIMO-VLC systems to have better useful insights.**

***Index Terms*—Diversity-multiplexing tradeoff (DMT), multiple-input multiple-output (MIMO), single-input single-output (SISO), visible light communication (VLC).**

I. Introduction

The ever-increasing demand for smartphones and information data applications, such as video streaming and cloud computing, wireless data traffic is anticipated to show a drastic increase by over a factor of 200, from 3 exabytes in 2010 to over 1000 exabytes by 2030 [1]. Optical wireless communication (OWC) is a modern communication technology that has attracted a significant attention in the research community. Unlike the radio-frequency (RF) communication technology, OWC systems are entirely independent of spectrum licensing. Due to this, OWC systems offer high data-rates and very secured transmission. OWC based systems find its applications in many areas, such as indoor communication, terrestrial outdoor communication, satellite-based communication, etc. [2–4]. OWC is anticipated to be an essential alternative for future low-mobility indoor wireless communications. As far as indoor OWC is concerned, it includes two main technologies: infrared communications and visible light communications (VLC). The indoor OWC system, also known as VLC, corresponds to the transmission of the optical signal from the visible spectrum (400 THz to 800 THz). For illumination and data communications, VLC systems often employ light-emitting diodes (LEDs) [3–6]. The transmission and reception of the data take place through LEDs and photodetectors, respectively. Since LED can switch faster than the human eye can respond, they can modulate at high data rates; offering not only illumination but also communication capabilities. Due to the widespread deployment of LEDs for energy conservation and convergence of illumination and communications, nowadays, VLC is considered as one of the most vital green communication technologies [7].

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Wireless communication systems employing multiple-input multiple-output (MIMO) technology has fascinated more and more researchers over the last decade. As far as the reliability of the communication system is concerned, MIMO systems offer superior performance as compared to the conventional single-input single-output (SISO) systems in terms of the availability of independent propagation paths. These independent propagation paths are characterized by the term ‘diversity gain’. In simple words, the term ‘diversity’corresponds to the reliability of the communication link. Further, MIMO systems utilize parallel spatial modes characterized by the term ‘spatial multiplexing gain’or simply ‘multiplexing gain’. However, it is proven in [8], [9] that there is a fundamental tradeoff between the diversity and multiplexing gains, known as diversity-multiplexing tradeoff (DMT). Over the past few years, VLC-based studies with different aspects into consideration including channel modeling [6], modulation [10], coding [11], MIMO systems [12], performance evaluation of non-orthogonal multiple access (NOMA) [13], transceiver design [14] and channel capacity [15] have been a part of the research.

Most of the literature on DMT mainly emphasizes on the RF systems, and for which the channel coefficients are complex. On the other side, applying the concept of DMT in VLC systems (for which the channel coefficients are real and positive) requires careful reconsiderations. It is mainly because of the following reasons: (1) The channel characteristics play a significant role when the concept of DMT, is applied to indoor VLC systems; and (2) apart from the channel characteristics, there are multiple parameters involved in the VLC systems that require careful attention in deciding which parameters will affect the optimal DMT. Further, to the best of our knowledge, no study on DMT in VLC-based communication systems is done in the literature. Thus it motivates us to contribute to the existing research by giving some novel insights into the study of DMT-based VLC systems. In this paper, our major contributions and observations are:

- A novel investigation of the optimal DMT for an indoor environment based VLC system model is performed.

- The optimal DMT for SISO-VLC and MIMO-VLC systems is analyzed, considering a practical transmission scenario, i.e., there is a possibility that the VLC link may undergo outage.
- Novel analytical expressions for the probability density function (PDF), outage probability (in terms of the multiplexing gain and other VLC parameters) and outage diversity order for the SISO-VLC and MIMO-VLC systems are derived.
- From the numerical results, it is observed that the MIMO-VLC system provides better optimal DMT as compared to the SISO-VLC system for different indoor scenarios.

II. System Model

Let there be N transmitting LEDs and M end-users in a room. The N transmitting LEDs are positioned on the ceiling of the room to transmit information data to the end-users. A particular transmitting LED is denoted by l_j, $j = 1, 2, \cdots, N$ and a particular end-user is denoted by u_i, $i = 1, 2, \cdots, M$. As shown in Fig. 1, the LED transmitter is placed at a height D (in metres) from the u_i end-user with angle of irradiance ϕ_{u_i}, the angle of incidence ψ_{u_i} and radius r_{u_i} on the polar coordinate plane. The euclidean distance between the l_j^{th} LED and the u_i^{th} photodetector receiver is denoted by d_{u_i}.

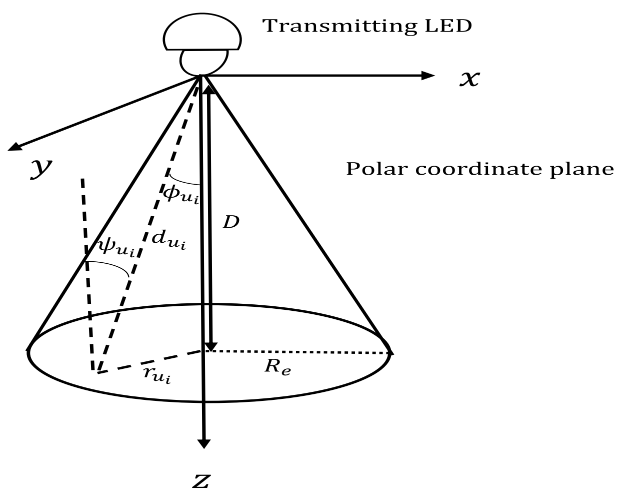

Fig. 1. Geometry of the indoor line-of-sight (LOS) VLC propagation model.

The intercell interferences received from other transmitting LEDs are approximated with the additive Gaussian process [6], [12]. Further, the semi-angle at the half-illuminance of LED (half-power angle), $\Phi_{\frac{1}{2}}$, can be mathematically related to the maximum radius of a LED cell footprint, R_e, as $R_e = \frac{D\sin\left(\Phi_{\frac{1}{2}}\right)}{\cos\left(\Phi_{\frac{1}{2}}\right)}$. The order of Lambertian emission (mode number expressing directivity of the source beam) is defined as [3], [6]:

$$K_o \triangleq -\frac{\ln 2}{\ln\left(\cos\left(\Phi_{\frac{1}{2}}\right)\right)}, \tag{1}$$

where $0^{\circ} < \Phi_{\frac{1}{2}} < 90^{\circ}$. The channel coefficient of the LOS link between LED and end-user u_i is given by [6]:

$$h_{u_i} = \frac{A_d(K_o+1)\Re}{2\pi d_{u_i}^2}\cos^{K_o}(\phi_{u_i})Tg(\psi_{u_i})\cos(\psi_{u_i}), \tag{2}$$

where A_d is a physical surface area of the photodetector, $\Re$ is the photdetector responsivity, T is the gain of the optical filter, and $g(\psi_{u_i})$, the optical concentrator, is defined as [11]:

$$g(\psi_{u_i}) \triangleq \begin{cases} \frac{n^2}{\sin^2(\Psi)}, & 0 \le \psi_{u_i} \le \Psi \\ 0, & \psi_{u_i} > \Psi \end{cases}, \tag{3}$$

where n is the refractive index of lens at a photodetector and $\Psi \le \frac{\pi}{2}$ is the field-of-view (FOV) of the photodetector receiver. It is assumed that the surface of the photodetector receiver is parallel to the ground and the photodetector receiver has no orientation towards the LED. Thus, it holds that $\phi_{u_i} = \psi_{u_i}$.

Based on the geometry, as shown in the figure, the following relations hold:

$$d_{ui} = \sqrt{r_{u_i}^2 + D^2}, \tag{4a}$$

$$\cos(\phi_{u_i}) = \frac{D}{\sqrt{r_{u_i}^2 + D^2}}, \tag{4b}$$

$$\xi = \frac{A_d(K_o+1)\Re}{2\pi}Tg(\psi_{u_i})D^{K_o+1}. \tag{4c}$$

Accordingly, the channel coefficient h_{u_i} in (2) can be rewritten as:

$$h_{ui} = \frac{\xi}{(r_{u_i}^2 + D^2)^{\frac{K_o+3}{2}}}. \tag{5}$$

Assuming that the user position is random over the circular area, the radial distance of the u_i^{th} end-user is modeled by a uniform distribution with the PDF:

$$f_{r_{u_i}}(r_{u_i}) = \frac{2r_{u_i}}{R_e^2}, \tag{6}$$

where $0 \le r_{u_i} \le R_e$.

Moreover, the instantaneous signal-to-noise ratio (SNR) of the VLC channel of the u_i^{th} end-user is defined as:

$$\delta \triangleq \frac{\eta^2 h_{u_i}^2 P_E^2}{\sigma_n^2}, \tag{7}$$

where η is the optical-to-electrical conversion efficiency, P_E is the transmitted optical intensity of each LED and σ_n^2 represents the noise variance.

Remark 1: *A complete VLC channel comprises not only the LOS link, but also the diffused components arising due to the light reflections from interior surfaces. However, it is reported in [11] that the strongest diffused component is at least 7 dB (electrical) lower than the weakest LOS component. Therefore, only the LOS link is considered in our analytical framework.*

III. Fundamental Definitions

In this section, we introduce some important definitions and notations that will be used throughout this paper.

1. For any function of δ, i.e., $f(\delta)$, the following equality:
$$\lim_{\delta\to\infty}\frac{\log f(\delta)}{\log\delta}=p, \tag{8}$$
is denoted as $f(\delta)\doteq\delta^p$, where $\doteq$ represents exponential equality.
2. An outage event has the interpretation that the channel does not support a target data rate, or in other words, when the instantaneous channel capacity, $C(\mathbf{H})$, is less than the target data rate, $R(\delta)$. Mathematically,
$$P_{out}(R,\delta)\triangleq\Pr(C(\mathbf{H})<R(\delta)), \tag{9}$$
where $P_{out}(R,\delta)$ is the outage error probability and the probability is defined over the random channel matrix $\mathbf{H}$. In this paper, we use Telatar's capacity formula, which is used to find capacity of any wireless communication system and is given by [16], [17]:
$$\boldsymbol{C}(\mathbf{H})\triangleq\frac{1}{2}\log_2\left(\det\left(\boldsymbol{I}_M+\frac{\delta}{N^2}\mathbf{H}\mathbf{H}^{\mathrm{T}}\right)\right), \tag{10}$$
where $\det(.)$ denotes the determinant, the identity matrix of order $M\times M$ is denoted as $\mathbf{I}_M$ and $\mathbf{H}^{\mathrm{T}}$ denotes the transpose of matrix $\mathbf{H}$. Since the channel coefficients are real and positive in intensity modulation and direct detection (IM/DD) VLC system, hence (10) contains $\mathbf{H}^{\mathrm{T}}$ instead of $\mathbf{H}^{\dagger}$, where $\mathbf{H}^{\dagger}$ denotes hermitian of the matrix $\mathbf{H}$.

Since the capacity expression given in (10) corresponds to an IM/DD VLC system, the expression in (10) is slightly different from that given in [16], [17]. The first difference is the factor $1/2$ due to half degrees of freedom offered by the real-valued channel gains as compared to the complex-valued channel gains. The second difference is the term N^2; this is because the received electrical signal is directly proportional to the optical power in the VLC system. Moreover, the maximization of the capacity is done over the total transmit power constraint, which depends upon the peak power of the input signal and it is positive for an IM/DD VLC system.

3. Let d and r denote the diversity and multiplexing gains, respectively, then
$$d=-\lim_{\delta\to\infty}\frac{\log P_{out}(R,\delta)}{\log\delta}, \tag{11}$$
and
$$r=\lim_{\delta\to\infty}\frac{R(\delta)}{\frac{1}{2}\log\delta}. \tag{12}$$

IV. Outage Probability And Outage Diversity Order for SISO-VLC System

In this section, we will derive the outage probability and outage diversity order for the channel gain given by (5). For a SISO-VLC system, the channel capacity expression given in (10) conditioned on h_{u_i} can be written as:
$$C(h_{u_i})=\frac{1}{2}\log(1+\delta h_{u_i}^2). \tag{13}$$
From (9) and (12), the outage probability at very high SNR is obtained as:
$$\begin{aligned}P_{out}(R,\delta)&=\Pr\left(\frac{1}{2}\log(1+\delta h_{u_i}^2)\leq\frac{r}{2}\log(\delta)\right)\\&\approx\Pr\left(h_{u_i}\leq\delta^{\frac{-(1-r)}{2}}\right).\end{aligned} \tag{14}$$
Now, in order to compute (14), we need to determine the PDF of h_{u_i}. By utilizing the PDF of r_{u_i} given by (6), the PDF of h_{u_i} can be derived by applying the concept of transformation of random variables to (5):
$$\begin{aligned}f_{h_{u_i}}(h_{u_i})&=\frac{2}{R_e^2(K_o+3)}\xi^{\frac{2}{K_o+3}}h_{u_i}^C\\&=Bh_{u_i}^C.\end{aligned} \tag{15}$$
In (15), $h_{u_i}\in[h_{u_i\min},\ h_{u_i\max}]$, where $h_{u_i\min}=\frac{\xi}{(R_e^2+D^2)^{\frac{K_o+3}{2}}}$, $h_{u_i\max}=\frac{\xi}{D^{K_o+3}}$, $B=\frac{2}{R_e^2(K_o+3)}\xi^{\frac{2}{K_o+3}}$, and $C=\frac{K_o+5}{K_o+3}$, such that the optimal outage diversity order $d_{out}^*(r)$ is non-negative. Therefore, from (14) we get:
$$\begin{aligned}P_{out}(R,\delta)&=\int_0^{\delta^{\frac{-(1-r)}{2}}}f_{h_{u_i}}(h_{u_i})dh_{u_i}\\&=\frac{B}{1+C}\left[\delta^{-(1+C)\frac{(1-r)}{2}}\right].\end{aligned} \tag{16}$$
Further, using (8), (16) can be written as:
$$P_{out}(R,\delta)\doteq\delta^{-(1+C)\frac{(1-r)}{2}}. \tag{17}$$

On substituting (17) into (11), the optimal outage diversity order is obtained as:
$$d_{out}^*(r)=(1+C)\frac{(1-r)}{2}, \tag{18}$$
where $r\in(0,m)$. Further, we define $t\triangleq\max(N,M)$ and $m\triangleq\min(N,M)$. Note that m is the maximum multiplexing gain in the VLC channel. For a SISO-VLC system, $r\in(0,1)$. The derived expression in (18), will be used as a benchmark for verifying the correctness of the outage diversity order for the MIMO-VLC system in the subsequent section.

V. Outage Probability And Outage Diversity Order for MIMO-VLC System

In this section, we will derive the outage probability and the outage diversity order for a MIMO-VLC system. For a general $N\times M$ MIMO-VLC system, where N being the number of transmitting LEDs and M corresponds to the number of photodetectors, outage occurs when the channel matrix $\mathbf{H}$ is nearly singular. Therefore, we must focus on the probability that the singular values of $\mathbf{H}$ are close to zero. On substituting

(10) into (9) and applying eigenvalue decomposition (EVD) of $\mathbf{HH}^{\mathrm{T}}$, we have:

$$P_{out}(R,\delta) = \Pr\left(\sum_{k=1}^{m}\log\left(1+\frac{\delta}{N}\lambda_k\right) < \delta^r\right), \tag{19}$$

where λ_k, $k = 1, 2, \cdots, m$, are real and non-negative eigenvalues of $\mathbf{HH}^{\mathrm{T}}$ and $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_m$. Let us define $\lambda_i \triangleq \delta^{-\alpha_i}$ [8]. Therefore, at high SNR, $(1+\frac{\delta}{N}\lambda_i) \doteq \delta^{(1-\alpha_i)^+}$, where x^+ denotes max(0,x). Here, α_i indicates the level of the singularity of channel matrix $\mathbf{H}$. Further, larger the $\alpha_i's$ are, the more singular $\mathbf{H}$ is. Thus, (19) can be rewritten at high SNR as:

$$P_{out}(R,\delta) \doteq \Pr\left[\sum_{i=1}^{m}(1-\alpha_i)^+ < r\right], \tag{20}$$

where $\mathcal{A} = \left\{\alpha : \alpha_1 \geq \cdots \geq \alpha_m \geq 0, \sum_{i=1}^{m}(1-\alpha_i)^+ \leq r\right\}$ describes the outage event in terms of singularity levels. The outage probability can be computed as the probability that $\alpha \in \mathcal{A}$:

$$P_{out}(R,\delta) \doteq \int_{\mathcal{A}} p(\alpha)d\alpha, \tag{21}$$

where $p(\alpha)$ represents the joint PDF of $\alpha_i's$. The PDF of α derived in [18] is valid for the complex channel gain matrix $\mathbf{H}$. However, the PDF of α in terms of the PDF of $\mathbf{H}$, when $\mathbf{H}$ is real, is derived as:

$$\begin{aligned} p_{(\alpha)}(\alpha) &= K_{N,M}(\log\delta)^m\left(\prod_{i=1}^{m}\delta^{\frac{-(t-m+1)\alpha_i}{2}}\right) \\ &\times\prod_{i<j}|\delta^{-\alpha_i}-\delta^{-\alpha_j}|\times\left[\int_{\mathbf{V}_{M,M}}\int_{\mathbf{V}_{M,N}}p_{\mathbf{H}(\mathbf{U\ D\ Q})}d\mathbf{Q}d\mathbf{U}\right], \end{aligned} \tag{22}$$

where $K_{N,M}$ is normalization constant, $\mathbf{D}$= $diag\left\{\delta^{-\alpha_1/2},\cdots,\delta^{-\alpha_m/2}\right\}$, $diag(.)$ denotes the diagonal, $\mathbf{V}_{M,M}$ and $\mathbf{V}_{M,N}$ are the Stiefel manifolds [19]. On substituting (22) into (21), the outage probability can be written as:

$$\begin{aligned} P_{out}(R,\delta) &\doteq \int_{\mathcal{A}}K_{N,M}(\log\delta)^m\left(\prod_{i=1}^{m}\delta^{\frac{-(t-m+1)\alpha_i}{2}}\right) \\ &\times\prod_{i<j}|\delta^{-\alpha_i}-\delta^{-\alpha_j}|\times\left[\int_{\mathbf{V}_{M,M}}\int_{\mathbf{V}_{M,N}}p_{\mathbf{H}(\mathbf{U\ D\ Q})}d\mathbf{Q}d\mathbf{U}\right]. \end{aligned} \tag{23}$$

Since we are interested only in the SNR exponent of the $P_{out}(R,\delta)$, i.e., $\lim_{\delta\to\infty}\log P_{out}(R,\delta)/log(\delta)$, we can make some approximations in the integral given in (23). Firstly, the term $K_{N,M}(\log\delta)^m$ has no influence on the δ exponent. Secondly, for any $j > i$, $|\delta^{-\alpha_i} - \delta^{-\alpha_j}| \doteq \delta^{-\alpha_j}$, where $\alpha_j < \alpha_i$. Therefore, after considering these approximations, (23) can be rewritten as:

$$\begin{aligned} P_{out}(R,\delta) &\doteq \int_{\mathcal{A}}\delta^{-\sum_{i=1}^{m}\frac{t-m+2i-1}{2}\alpha_i} \\ &\times\left[\int_{\mathbf{V}_{M,M}}\int_{\mathbf{V}_{M,N}}p_{\mathbf{H}(\mathbf{U\ D\ Q})}d\mathbf{Q}d\mathbf{U}\right]. \end{aligned} \tag{24}$$

Moreover, the channel gains are independent to each other, hence:

$$\begin{aligned} p_{\mathbf{H}}(\mathbf{H}) &= \prod_{i=1}^{M}\prod_{j=1}^{N}f_{h_{ij}}(h_{ij}) \\ &= \left(\frac{2}{R_e^2(K_o+3)}\xi^{\frac{2}{K_o+3}}\right)^{NM}\prod_{i=1}^{M}\prod_{j=1}^{N}h_{ij}^{C}, \end{aligned} \tag{25}$$

where the PDF $f_{h_{ij}}(h_{ij})$ is given by (15). Further, we need to find the exponent of the δ of the term $f(\delta)$= $\left[\int_{\mathbf{V}_{M,M}}\int_{\mathbf{V}_{M,N}}p_{\mathbf{H}(\mathbf{U\ D\ Q})}d\mathbf{Q}d\mathbf{U}\right]$ (given in (24)). Since we are interested in the near zero behaviour of the PDF of $\mathbf{H}$ to compute the outage diversity order, the PDF of $\mathbf{H}$ in terms of the polynomial of δ is given by [17, Eq.(19)]. We must note that h_{ij}= $[\mathbf{U\ D\ Q}]_{ij}$ is a polynomial expression of δ [18], and the coefficients are functions of $\mathbf{U}$ and $\mathbf{Q}$. The term with the highest order is $\delta^{\frac{-\alpha_m}{2}}$ and its corresponding coefficient is nonzero. Accordingly, for high SNR ($\delta \to \infty$), we can write $h_{ij} \doteq \delta^{\frac{-\alpha_m}{2}}$. The term $\left(\frac{2}{R_e^2(K_o+3)}\xi^{\frac{2}{K_o+3}}\right)^{NM}$ can be neglected at high SNR, since it does not influence the exponent of δ. Therefore, we have $f(\delta) \doteq \delta^{-\frac{MN\alpha_m C}{2}}$. On substituting $f(\delta)$ into (24), the outage probability is obtained as:

$$P_{out}(R,\delta) \doteq \delta^{-\inf_{\alpha\in\mathcal{A}}\left\{\sum_{i=1}^{m}\frac{t-m+2i-1}{2}\alpha_i+\frac{NM\alpha_m C}{2}\right\}} \tag{26}$$

On substituting (26) into (11), the optimal outage diversity order for the MIMO-VLC system is given as:

$$d^*_{out}(r) = \inf_{\alpha\in\mathcal{A}}\left\{\sum_{i=1}^{m}\frac{t-m+2i-1}{2}\alpha_i+\frac{NM\alpha_m C}{2}\right\}, \tag{27}$$

where $\mathcal{A} = \left\{\alpha : \alpha_1 \geq \cdots \geq \alpha_m \geq 0, \sum_{i=1}^{m}(1-\alpha_i)^+ \leq r\right\}$ describes the outage event in terms of singularity levels. Further, to check the correctness of the derived outage diversity order expression given by (27), we substitute $N = M = 1$ into (27), which reduces it into an expression for the outage diversity SISO-VLC system given by (18).

Remark 2**:** *To obtain the optimal $\alpha_i's$, we solve the optimization problem subject to the constraints defined in the set $\mathcal{A}$. This can be done with the help of MATLAB software.*

VI. Numerical Results And Discussions

In this section, we present and discuss the numerical results derived in the previous sections. Moreover, we will compare and investigate which system provides the optimal DMT.

In Fig. 2, we plot in the outage performance plots for a 4×4 MIMO-VLC system. The main objective of giving these

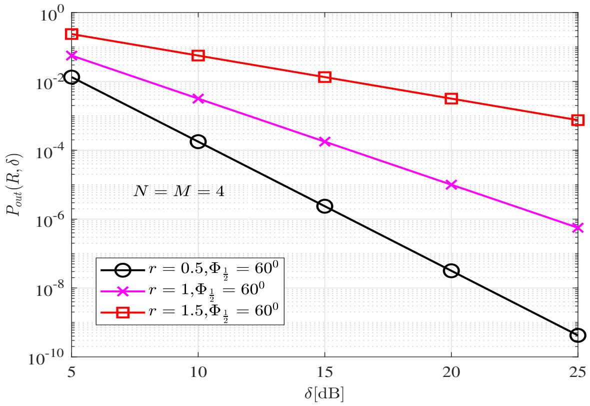

Fig. 2. Asymptotic outage probability versus SNR plots for $\Phi_{\frac{1}{2}} = 60^{\circ}$ and different multiplexing gains for a 4×4 MIMO-VLC system.

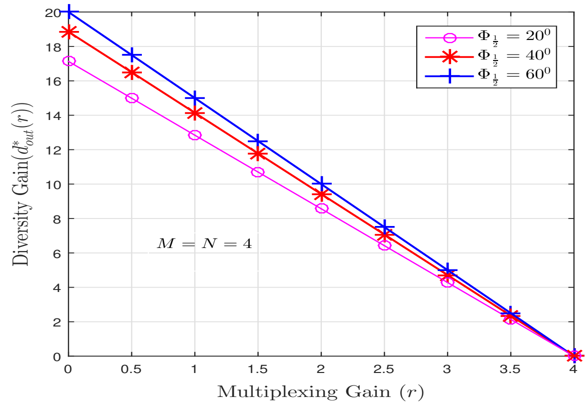

Fig. 4. Optimal DMT curves for 4×4 MIMO-VLC system.

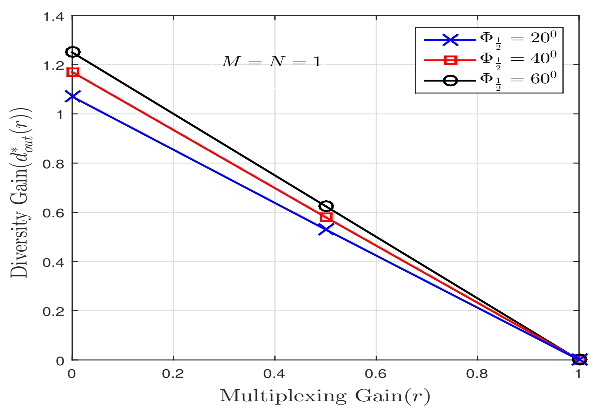

Fig. 3. Optimal DMT curves for SISO-VLC system.

plots is to show the outage performance for different indoor VLC scenarios. We use (26) to obtain the outage probability versus SNR plots. It can be seen from (26) that the outage probability mainly depends on the parameters, such as half-power angle, multiplexing gain and the optimal $\alpha_i' s$ of the 4×4 MIMO channel matrix. To obtain the optimal $\alpha_i' s$, we solve the optimization problem subject to the constraints defined in the set $\mathcal{A}$. This can be done with the help of MATLAB software. It can be clearly seen from the figure that as the multiplexing gain increases, the outage performance deteriorates.

Figure 3 shows the optimal DMT curves for a SISO-VLC ($M = N = 1$) system. We use (18) to plot these optimal DMT curves. From (18), it is clear that the optimal outage diversity gain, $d^*_{out}(r)$ is dependent on the multiplexing gain (r) and the order of Lambertian emission (K_o). However, the latter instead depends on the half-power angle ($\Phi_{\frac{1}{2}}$) of the LED transmitter. Therefore, we provide the optimal DMT curves for different values of $\Phi_{\frac{1}{2}}$. For $\Phi_{\frac{1}{2}} = 20^{\circ}$, the optimal outage diversity gain, $d^*_{out}(0) \approx 1$, is attained at a minimum multiplexing gain ($r = 0$). However, the optimal outage diversity gain, $d^*_{out}(1) = 0$, at a maximum multiplexing gain ($r = m = 1$) for a SISO-VLC system. This shows the optimal tradeoff between the diversity gain and the multiplexing gain. For $\Phi_{\frac{1}{2}} = 40^{\circ}$ and $\Phi_{\frac{1}{2}} = 60^{\circ}$, $d^*_{out}(0) \approx 1.2$ and 1.25, respectively.

Observation 1: *The slight increase in the optimal diversity gain for a SISO-VLC system is due to the increase in the half-power angle ($\Phi_{\frac{1}{2}}$) of the LED transmitter.*

In Fig. 4, the optimal DMT curves are plotted for a 4×4 MIMO-VLC system. We use (27) to plot these optimal DMT curves. From (27), it is clear that the optimal outage diversity gain $d^*_{out}(r)$ not only depends on the half-power angle ($\Phi_{\frac{1}{2}}$), but also the optimal $\alpha_i' s$ of the channel matrix. For $\Phi_{\frac{1}{2}} = 20^{\circ}$, the optimal outage diversity gain, $d^*_{out}(0) \approx 17$, is attained at a minimum multiplexing gain ($r = 0$), as shown in Fig. 4. However, the optimal outage diversity gain of $d^*_{out}(4) = 0$ is obtained at a maximum multiplexing gain ($r = m = 4$) for a 4×4 MIMO-VLC system. Further, it can be seen from the figure that for $\Phi_{\frac{1}{2}} = 40^{\circ}$ and $\Phi_{\frac{1}{2}} = 60^{\circ}$, $d^*_{out}(0) \approx 19$ and 20, respectively.

Observation 2: *The optimal DMT corresponding to 4×4 MIMO-VLC system is significantly better as compared to the SISO-VLC system.*

Figure 5 shows optimal DMT curves for an 8×8 MIMO-VLC system. It can be seen from the figure that for $\Phi_{\frac{1}{2}} = 20^{\circ}$, the optimal outage diversity gain of $d^*_{out}(0) \approx 70$, is attained at a minimum multiplexing gain ($r = 0$). However, the optimal outage diversity gain, $d^*_{out}(8) = 0$, is attained at a maximum multiplexing gain ($r = m = 8$) for a 8×8 MIMO-VLC system. Further, for $\Phi_{\frac{1}{2}} = 40^{\circ}$ and $\Phi_{\frac{1}{2}} = 60^{\circ}$, it is shown in the figure that $d^*_{out}(0) \approx 75$ and 80, respectively. As far as the comparison of optimal DMT of 8×8 and 4×4 MIMO-VLC system is concerned, the 8×8 MIMO-VLC system provides a significant improvement in the optimal DMT.

Observation 3: *The significant improvement in the optimal DMT of an 8×8 MIMO-VLC system is due to the larger number of independent propagation paths. Besides, the dependency of the outage diversity gain on the half-power angle of the LED transmitter and the optimal $\alpha_i' s$ of the 8×8 MIMO channel matrix is another reason for this significant improvement.*

In Fig. 6, the optimal DMT curves are plotted for a 2×4 ($N \times M$) MIMO-VLC system. It can be seen from the figure that for $\Phi_{\frac{1}{2}} = 20^{\circ}$, the optimal outage diversity gain of $d^*_{out}(0) \approx 8.5$, is attained at a minimum multiplexing gain ($r = 0$). However, the optimal outage diversity gain, $d^*_{out}(2) =$

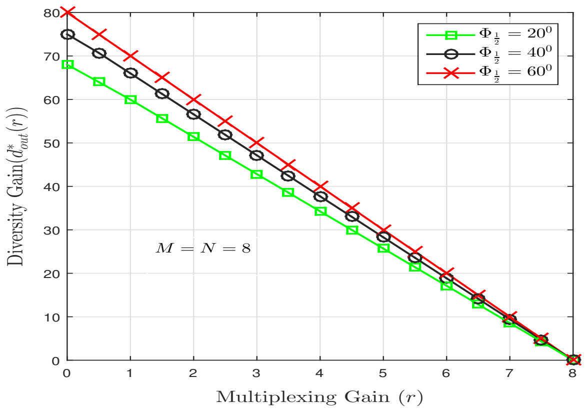

Fig. 5. Optimal DMT curves for 8×8 MIMO-VLC system.

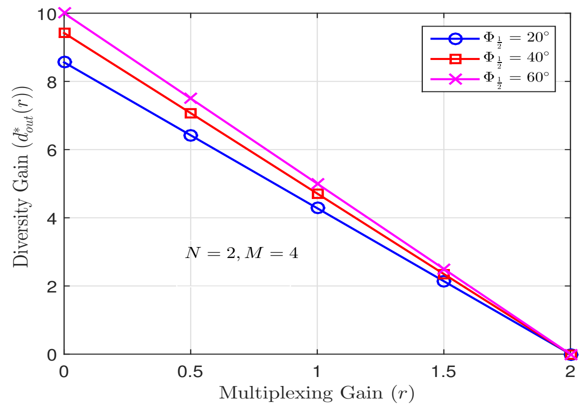

Fig. 6. Optimal DMT curves for 2×4 $(N \times M)$ MIMO-VLC system.

0, is attained at a maximum multiplexing gain ($r = m = 2$) for a 2×4 MIMO-VLC system. Further, for $\Phi_{\frac{1}{2}} = 40^\circ$ and $\Phi_{\frac{1}{2}} = 60^\circ$, it is shown in the figure that $d^*_{out}(0) \approx 9.5$ and 10, respectively.

***Observation 4*: *If we compare the DMT performance curves of 2×4 $(N\times M)$ and 4×4 $(M\times M)$ MIMO-VLC systems, it is observed that 4×4 MIMO-VLC system provides significantly better DMT performance. It is mainly because of the larger number of independent propagation paths offered by a 4×4 MIMO-VLC system. Note that the outage diversity gains corresponding to different values of $\Phi_{\frac{1}{2}}$ for a 2×4 MIMO-VLC system are almost half as compared to the 4×4 MIMO-VLC system.*

VII. Conclusion

The optimal DMT for an indoor environment based SISO-VLC and MIMO-VLC systems has been investigated. Novel analytical expressions for the PDF, outage probability (in terms of the multiplexing gain and other VLC parameters), and outage diversity order for the SISO-VLC and MIMO-VLC systems have been derived. From the numerical results, we have observed that the optimal DMT for MIMO-VLC systems is significantly better as compared to the SISO-VLC systems. This is mainly because of two reasons. Firstly, the MIMO-VLC system offers multiple independent propagation paths. Secondly, the optimal outage diversity gain in the case of the MIMO-VLC system not only depends on the half-power angle of the LED transmitter, but also the optimal singular values of the channel matrix.

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