

VFD-NOMA with Improper Gaussian Signaling Under Imperfect SIC and Inter-Relay Interference

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Abstract—This paper investigates the performance of Improper Gaussian Signaling (IGS) in a Virtual Full-Duplex Non-Orthogonal Multiple Access (VFD-NOMA) network over Nakagami- m fading channels. Unlike conventional Proper Gaussian Signaling (PGS), IGS exploits asymmetric signal statistics through a non-zero complementary variance, offering added flexibility to mitigate interference. We consider a network comprising two relay users with direct links to the source and one destination user without a direct link. The achievable rates are characterized as a function of SNR for two distinct user locations, under both with and without inter-relay interference (IRI) conditions. The results show that IGS outperforms PGS for the first relay user at high SNR. In contrast, PGS is more favorable for the second relay user under IRI and for the destination user across all scenarios, due to the compatibility of circular Gaussian statistics. The outage performance as well as the effect of varying threshold rates on outage behavior are also examined. Overall, the findings suggest that a hybrid IGS/PGS strategy, adapted to node role and interference conditions, can yield a more robust and spectrally efficient VFD-NOMA design.

Index Terms—Non-orthogonal multiple access, Improper Gaussian signaling, virtual full-duplex, inter-relay interference, successive interference cancellation.

I. INTRODUCTION

The relentless surge in demand for ultra-high data rates, massive machine-type connectivity, and superior spectral efficiency (SE) has positioned next-generation wireless networks at the forefront of telecommunications research. To meet these stringent requirements, non-orthogonal multiple access (NOMA) has emerged as a cornerstone technology [1], [2]. Employing power-domain multiplexing, NOMA allows multiple users to share identical time-frequency resources, significantly outperforming traditional orthogonal multiple access (OMA) in terms of user density and throughput [1]. However, the practical deployment of NOMA is hindered by its reliance on successive interference cancellation (SIC) [3].

Recent literature [4] has identified Improper Gaussian Signaling (IGS) as a powerful tool for managing such residual interference. This added degree of freedom allows the transmitter to shape interference, making it more manageable for receivers in interference-limited environments. For instance, the work in [5] demonstrated that IGS significantly improves rate performance in downlink NOMA systems subject to imperfect SIC.

To further push the boundaries of SE, virtual full-duplex NOMA (VFD-NOMA) has been introduced [6]–[8]. VFD-

NOMA achieves the benefits of full-duplex communication by coordinating two half-duplex users to act as mutual relays, thereby bypassing the need for expensive self-interference cancellation hardware.

Despite the promising individual benefits of IGS and VFD-NOMA, several important challenges remain unresolved. Existing IGS-based NOMA studies primarily focus on conventional uplink and downlink transmission scenarios [4], [9], [10] and generally neglect cooperative virtual full-duplex architectures. Conversely, existing VFD-NOMA investigations [7], [8], [11] predominantly employ proper Gaussian signaling and do not exploit signal impropriety as an interference-management mechanism. More importantly, the inter-relay interference (IRI) inherent in VFD operation introduces a fundamentally different interference structure that is not captured by existing IGS-NOMA models. Consequently, the combined impact of signal impropriety, residual SIC, and IRI on the performance of cooperative VFD-NOMA systems remains insufficiently understood. To the best of the authors' knowledge, a comprehensive analytical study addressing these aspects has not yet been reported in the open literature.

The key contributions of this work are summarized as follows:

- We develop an analytical framework for an IGS-enabled cooperative VFD-NOMA system that jointly considers signal impropriety, residual SIC, and virtual full-duplex relaying.
- We investigate two practically relevant operating scenarios, namely VFD-NOMA without IRI and VFD-NOMA with IRI, thereby quantifying the impact of inter-relay interference on the effectiveness of improper signaling.
- Closed-form expressions for the achievable rate are derived for both scenarios. In addition, closed-form outage probability expressions are obtained for the case without IRI, providing useful performance benchmarks.
- For the IRI-limited scenario, the achievable-rate analysis reveals the complex coupling between signal impropriety and inter-relay interference.
- Numerical results provide insights into the conditions under which IGS offers measurable performance gains over conventional PGS in cooperative VFD-NOMA systems and demonstrate the influence of residual SIC and IRI on system performance.

The remainder of this paper is organized as follows: Section II describes the system model. Section III presents the outage probability analysis. Numerical results and discussions are provided in Section IV, and the paper is concluded in Section V.

II. SYSTEM MODEL

A. Preliminaries

Let x be a zero-mean complex random variable characterized by its variance and complementary variance, defined respectively as $\sigma_x^2 \triangleq \mathbb{E}(|x|^2)$ and $\tilde{\sigma}_x^2 \triangleq \mathbb{E}(x^2)$. The random variable x is said to be proper if its complementary variance vanishes, i.e., $\tilde{\sigma}_x^2 = 0$. Otherwise, x is termed improper [5].

The pair $(\sigma_x^2, \tilde{\sigma}_x^2)$ constitutes a valid second-order statistical description if and only if $\sigma_x^2 > 0$ and $|\tilde{\sigma}_x^2| \leq \sigma_x^2$. To quantify the degree of impropriety, the circularity (or impropriety) coefficient of x is defined as $\kappa_x \triangleq |\tilde{\sigma}_x^2|/\sigma_x^2$ ($0 \leq \kappa_x \leq 1$), where $\kappa_x = 0$ corresponds to a proper signal, while $\kappa_x = 1$ represents a maximally improper signal.

B. Signal Transmission Framework

We consider a conventional downlink VFD-NOMA architecture as in [11], where a source node S communicates with a destination user D which is far away from S with no direct link between them due to strong fading. Two relay users R_1 and R_2 assist in forwarding information from S to D . The S to R_i channels, i.e., h_{sr_i} , $i \in \{1, 2\}$, R_i to R_{3-i} channels, i.e., $h_{r_i, r_{3-i}}$, and R_i to D channels, i.e., $h_{r_i d}$, $i \in \{1, 2\}$ are complex channel coefficients which undergo independent non-identically distributed Nakagami (Nak)- m fading [12]. For $i \in \{1, 2\}$, $|h_{sr_i}| \sim \text{Nak}(m_{sr_i}, \Omega_{sr_i})$, $|h_{r_i d}| \sim \text{Nak}(m_{r_i d}, \Omega_{r_i d})$, and $|h_{r_1 r_2}| \sim \text{Nak}(m_{r_1 r_2}, \Omega_{r_1 r_2})$. Here, we assume that the reciprocity holds for the inter-relay interference channel, $h_{r_1 r_2}$. To this end, we note that for any $|h| \sim \text{Nak}(m, \Omega)$, $|h|^2$ is Gamma distributed with shape m and scale Ω/m [12]. For ease of presentation, let $k_i = m_{sr_i}$, $\theta_i = \Omega_{sr_i}/m_{sr_i}$, $i \in \{1, 2\}$, $k_3 = m_{r_1 r_2}$, $\theta_3 = \Omega_{r_1 r_2}/m_{r_1 r_2}$, $k_4 = m_{r_1 d}$, $\theta_4 = \Omega_{r_1 d}/m_{r_1 d}$ and $k_5 = m_{r_2 d}$, $\theta_5 = \Omega_{r_2 d}/m_{r_2 d}$. Also, for any channel of the form h_{ab} , g_{ab} denotes $|h_{ab}|^2$. The noise at each receiver is additive white Gaussian noise (AWGN) with zero mean and variance σ^2 , i.e., $w \sim \mathcal{CN}(0, \sigma^2)$.

Let P_s denote the total transmit power at S . Power-domain NOMA is employed with allocation coefficients α and β for relay and destination users, respectively, such that $\alpha < \beta$ and $\alpha + \beta = 1$. $\rho_s = P_s/\sigma^2$ is transmit SNR. As per the NOMA terminology, because of no direct link between the source S and the destination D , D is referred to as weak (far) user and R_1/R_2 is considered as strong (near) user. Each relay first decodes D 's signal and then applies SIC to decode its own symbol. Residual interference due to imperfect SIC is modeled by $\lambda \in [0, 1]$, where $\lambda = 0$ corresponds to perfect SIC and $\lambda = 1$ to completely ineffective SIC.

The achievable rate associated with any decoding operation in the proposed VFD-NOMA system is evaluated using the generic IGS rate expression in (1). For a given received signal y and the corresponding interference-plus-noise component

z , the quantities required are the variance-absolute value of complementary variance pairs $(\sigma_y^2, |\tilde{\sigma}_y^2|)$ and $(\sigma_z^2, |\tilde{\sigma}_z^2|)$, respectively. Therefore, in subsequent derivations, only these quantities are specified for each decoding stage, while the achievable rate follows directly from (1) through the appropriate substitution.

$$\mathcal{R} = \frac{1}{6} \log_2 \frac{\sigma_y^4 - |\tilde{\sigma}_y^2|^2}{\sigma_z^4 - |\tilde{\sigma}_z^2|^2} \quad (1)$$

Following the IGS achievable-rate formulation in [5], [13], [14], the factor $1/6$ is obtained from the $1/2$ normalization inherent to IGS and the $1/3$ time-sharing factor of the considered three-phase VFD-NOMA protocol.

Conventional VFD-NOMA system works in three different phases as in [8].

1) *Phase-I*: The signal received by R_1 from S is,

$$y_{r_1} = h_{sr_1} \left(\sqrt{\alpha P_s} x_{r_1} + \sqrt{\beta P_s} x_d^I \right) + w_1 \quad (2)$$

where x_{r_1} is the transmitted signal of relay user R_1 , obeying the standard PGS, and x_d^I is the first transmitted signal of destination user D , subject to a potential IGS with impropriety degree $\kappa_1 \triangleq |\tilde{\sigma}_{x_d^I}^2|/\sigma_{x_d^I}^2$, in which $\tilde{\sigma}_{x_d^I}^2 \triangleq \mathbb{E}(x_d^I)^2$ and $\sigma_{x_d^I}^2 \triangleq \mathbb{E}(|x_d^I|^2)$. w_1 is AWGN at R_1 with zero mean and variance $\sigma_{w_1}^2$, i.e., $w_1 \sim \mathcal{CN}(0, \sigma_{w_1}^2)$. Moreover, let $z_1 = h_{sr_1} \sqrt{\alpha P_s} x_{r_1} + w_1$ represents the interference-plus-noise component while decoding x_d^I .

For the decoding of x_d^I at R_1 , the variance and absolute value of complementary variance terms required in (1) are

$$\sigma_{y_{r_1}}^2 = P_s g_{sr_1} + \sigma_{w_1}^2, \quad |\tilde{\sigma}_{y_{r_1}}^2| = \beta P_s g_{sr_1} \kappa_1 \quad (3)$$

$$\sigma_{z_1}^2 = \alpha P_s g_{sr_1} + \sigma_{w_1}^2, \quad |\tilde{\sigma}_{z_1}^2| = 0 \quad (4)$$

By substituting (3) and (4) into (1), the achievable rate at R_1 to decode x_d^I i.e. $\mathcal{R}_{r_1}^{d(I)}$ can be evaluated as

$$\mathcal{R}_{r_1}^{d(I)} = \frac{1}{6} \log_2 \left[1 + \frac{\beta^2 \rho_s^2 g_{sr_1}^2 (1 - \kappa_1^2)}{(1 + \alpha \rho_s g_{sr_1})^2} + \frac{2\beta \rho_s g_{sr_1}}{1 + \alpha \rho_s g_{sr_1}} \right] \quad (5)$$

Similarly, the signal at R_1 after SIC is,

$$y'_{r_1} = \sqrt{\alpha P_s} h_{sr_1} x_{r_1} + \lambda \sqrt{\beta P_s} h_{sr_1} x_d^I + w_1 \quad (6)$$

where, λ is imperfect SIC parameter.

For the decoding of x_{r_1} after SIC, the corresponding variance and absolute value of complementary variance terms are

$$\sigma_{y'_{r_1}}^2 = \alpha P_s g_{sr_1} + \lambda^2 \beta P_s g_{sr_1} + \sigma_{w_1}^2 \quad (7)$$

$$|\tilde{\sigma}_{y'_{r_1}}^2| = \lambda^2 \beta P_s g_{sr_1} \kappa_1 \quad (8)$$

$$\sigma_{z_1}^2 = \lambda^2 \beta P_s g_{sr_1} + \sigma_{w_1}^2 \quad (9)$$

$$|\tilde{\sigma}_{z_1}^2| = \lambda^2 \beta P_s g_{sr_1} \kappa_1, \quad (10)$$

Substituting the above quantities into (1) gives the achievable rate at R_1 to decode x_{r_1} i.e. $\mathcal{R}_{r_1}^{r_1}$ as,

$$\mathcal{R}_{r_1}^{r_1} = \frac{1}{6} \log_2 \left[1 + \frac{\alpha^2 \rho_s^2 g_{sr_1}^2 + 2\alpha \rho_s g_{sr_1} [1 + \lambda^2 \beta \rho_s g_{sr_1}]}{1 + 2\lambda^2 \beta \rho_s g_{sr_1} + \lambda^4 \beta^2 \rho_s^2 g_{sr_1}^2 [1 - \kappa_1^2]} \right] \quad (11)$$

During phase-I, R_2 also receives the same signal as R_1 . For the decoding of x_d^I at R_2 , the relevant variance-absolute value of complementary variance pairs are

$$\sigma_{y_{r_2}}^2 = P_s g_{sr_2} + \sigma_{w_2}^2, \quad |\tilde{\sigma}_{y_{r_2}}^2| = \beta P_s g_{sr_2} \kappa_1 \quad (12)$$

$$\sigma_{z_2}^2 = \alpha P_s g_{sr_2} + \sigma_{w_2}^2, \quad |\tilde{\sigma}_{z_2}^2| = 0, \quad (13)$$

Substituting these quantities into (1) yields the achievable rate for decoding x_d^I at R_2 i.e. $\mathcal{R}_{r_2}^{d(I)}$ as

$$\mathcal{R}_{r_2}^{d(I)} = \frac{1}{6} \log_2 \left[1 + \frac{\beta^2 \rho_s^2 g_{sr_2}^2 (1 - \kappa_1^2)}{(1 + \alpha \rho_s g_{sr_2})^2} + \frac{2\beta \rho_s g_{sr_2}}{1 + \alpha \rho_s g_{sr_2}} \right] \quad (14)$$

2) *Phase-II*: : The signal received by R_2 from S and R_1 is,

$$y_{r_2} = h_{sr_2} \left(\sqrt{\alpha P_s} x_{r_2} + \sqrt{\beta P_s} x_d^{II} \right) + \lambda' h_{r_1 r_2} \sqrt{P_r} x_d^I + w_2 \quad (15)$$

where x_{r_2} is the transmitted signal of relay user R_2 , obeying the standard PGS, and x_d^{II} is another transmitted symbol of destination user D , subject to a potential IGS with impropriety degree $\kappa_2 \triangleq |\tilde{\sigma}_{x_d^{II}}^2|/\sigma_{x_d^{II}}^2$, in which $\tilde{\sigma}_{x_d^{II}}^2 \triangleq \mathbb{E}(x_d^{II})^2$ and $\sigma_{x_d^{II}}^2 \triangleq \mathbb{E}(|x_d^{II}|^2)$. Also, λ' is the residual IRI (R-IRI) parameter introduced due to imperfect cancellation of x_d^I at R_2 [11]. P_r denotes the relay transmit power, and for the sake of fairness in total transmission power, $P_r = P_s/2$ when R_1 operates as a transmitter during phase-II. w_2 is AWGN at R_2 with zero mean and variance $\sigma_{w_2}^2$, i.e., $w_2 \sim \mathcal{CN}(0, \sigma_{w_2}^2)$. For the decoding of x_d^{II} at R_2 , the variance-absolute value of complementary variance terms required in (1) are

$$\sigma_{y_{r_2}}^2 = P_s g_{sr_2} + \lambda'^2 P_r g_{r_1 r_2} + \sigma_{w_2}^2 \quad (16)$$

$$|\tilde{\sigma}_{y_{r_2}}^2| \leq \beta P_s g_{sr_2} \kappa_2 + \lambda'^2 P_r g_{r_1 r_2} \kappa_1 \quad (17)$$

$$\sigma_{z_2}^2 = \alpha P_s g_{sr_2} + \lambda'^2 P_r g_{r_1 r_2} + \sigma_{w_2}^2 \quad (18)$$

$$|\tilde{\sigma}_{z_2}^2| = \lambda'^2 P_r g_{r_1 r_2} \kappa_1, \quad (19)$$

Here, we have used triangle inequality to obtain $|\tilde{\sigma}_{y_{r_2}}^2|$ in (17). Substituting the above expressions into (1) and applying the triangle inequality yields

$$\mathcal{R}_{r_2}^{d(II)} \geq \frac{1}{6} \log_2 \left(\frac{(A - B)(A + B)}{C^2 - \lambda'^4 \rho_r^2 g_{r_1 r_2}^2 \kappa_1^2} \right) \triangleq \mathcal{R}_{r_2}^{d(II)L} \quad (20)$$

where,

$$A \triangleq \rho_s g_{sr_2} + \lambda'^2 \rho_r g_{r_1 r_2} + 1, \quad (21)$$

$$B \triangleq \beta \rho_s g_{sr_2} \kappa_2 + \lambda'^2 \rho_r g_{r_1 r_2} \kappa_1, \quad (22)$$

$$C \triangleq \alpha \rho_s g_{sr_2} + \lambda'^2 \rho_r g_{r_1 r_2} + 1. \quad (23)$$

where, $\rho_r = \rho_s/2$.

The signal at R_2 after SIC is,

$$y'_{r_2} = \sqrt{\alpha P_s} h_{sr_2} x_{r_2} + \lambda \sqrt{\beta P_s} h_{sr_2} x_d^{II} + \lambda' h_{r_1 r_2} \sqrt{P_r} x_d^I + w_2 \quad (24)$$

For the decoding of x_{r_2} after SIC, the corresponding variance-absolute value of complementary variance quantities are

$$\sigma_{y_{r_2}}^2 = \alpha P_s g_{sr_2} + \lambda^2 \beta P_s g_{sr_2} + \lambda'^2 P_r g_{r_1 r_2} + \sigma_{w_2}^2 \quad (25)$$

$$|\tilde{\sigma}_{y_{r_2}}^2| = \lambda^2 \beta P_s g_{sr_2} \kappa_2 + \lambda'^2 P_r g_{r_1 r_2} \kappa_1 \quad (26)$$

$$\sigma_{z_2}^2 = \lambda^2 \beta P_s g_{sr_2} + \lambda'^2 P_r g_{r_1 r_2} + \sigma_{w_2}^2 \quad (27)$$

$$|\tilde{\sigma}_{z_2}^2| = \lambda^2 \beta P_s g_{sr_2} \kappa_2 + \lambda'^2 P_r g_{r_1 r_2} \kappa_1, \quad (28)$$

Substituting above quantities into (1) gives

$$\mathcal{R}_{r_2}^{r_2} = \frac{1}{6} \log_2 \left(\frac{\alpha \rho_s g_{sr_2} (2X + \alpha \rho_s g_{sr_2})}{X^2 - Y^2} \right) \quad (29)$$

where,

$$X \triangleq \lambda^2 \beta \rho_s g_{sr_2} + \lambda'^2 \rho_r g_{r_1 r_2} + 1, \quad (30)$$

$$Y \triangleq \lambda^2 \beta \rho_s g_{sr_2} + \lambda'^2 \rho_r g_{r_1 r_2} (\kappa_1 + \kappa_2). \quad (31)$$

Now, the received signal at D from R_1 is,

$$y_d^{(I)} = \sqrt{P_r} x_d^I h_{r_1 d} + w_d \quad (32)$$

where, w_d is AWGN at D with zero mean and variance $\sigma_{w_d}^2$, i.e., $w_d \sim \mathcal{CN}(0, \sigma_{w_d}^2)$.

For the reception of x_d^I at D , the required variance-absolute value of complementary variance quantities in (1) are

$$\sigma_{y_d}^2 = P_r g_{r_1 d} + \sigma_{w_d}^2, \quad |\tilde{\sigma}_{y_d}^2| = P_r g_{r_1 d} \kappa_1 \quad (33)$$

$$\sigma_{z_d}^2 = \sigma_{w_d}^2, \quad |\tilde{\sigma}_{z_d}^2| = 0, \quad (34)$$

The achievable rate of D to decode the symbol of its own, x_d^I in phase-II i.e. $\mathcal{R}_d^{(I)}$ can be evaluated as

$$\mathcal{R}_d^{(I)} = \frac{1}{6} \log_2 \left(\rho_r^2 g_{r_1 d}^2 (1 - \kappa_1^2) + 2\rho_r g_{r_1 d} + 1 \right) \quad (35)$$

3) *Phase-III*: The signal received at D from R_2 during phase-III is¹,

$$y_d^{(II)} = \sqrt{P_r} x_d^{II} h_{r_2 d} + w_d \quad (36)$$

Similar to $\mathcal{R}_d^{(I)}$ in (35), the achievable rate of D to decode its second symbol x_d^{II} in phase-III i.e. $\mathcal{R}_d^{(II)}$ is given by

$$\mathcal{R}_d^{(II)} = \frac{1}{6} \log_2 \left(\rho_r^2 g_{r_2 d}^2 (1 - \kappa_2^2) + 2\rho_r g_{r_2 d} + 1 \right) \quad (37)$$

The variance-absolute value of complementary variance pair expressions in this case are identical to those given in (33) and (34), respectively, with $g_{r_1 d}$ replaced by $g_{r_2 d}$.

The overall achievable rate at each node is obtained by summing the rates associated with the successfully decoded symbols at that node. Accordingly, the achievable rate at R_1 is given by $\mathcal{R}_{r_1}^{d(I)} + \mathcal{R}_{r_1}^{r_1}$, since R_1 first decodes the destination symbol x_d^I and then its own symbol x_{r_1} . Similarly, the achievable rate at R_2 is $\mathcal{R}_{r_2}^{d(II)} + \mathcal{R}_{r_2}^{r_2}$, where R_2 decodes both the destination symbol x_d^{II} and its own symbol x_{r_2} . At the destination, two symbols are received through the two relay paths; therefore, the overall achievable rate is $\min(\mathcal{R}_{r_1}^{d(I)}, \mathcal{R}_d^{(I)}) + \min(\mathcal{R}_{r_2}^{d(II)}, \mathcal{R}_d^{(II)})$.

¹Successive relaying can also be an option for phase-III but our focus is on the three phase VFD-NOMA [8], [11].

III. OUTAGE PROBABILITY ANALYSIS

In this section, we obtain outage probability expressions at all the three nodes R_1 , R_2 and D , considering IRI. Let $\mathcal{R}_1^{th}$, $\mathcal{R}_2^{th}$ and $\mathcal{R}_d^{th}$ be the target rates of R_1 , R_2 and D , respectively.

A. Outage Probability Analysis at Relay Users

The outage probability at R_1 , i.e. $\mathcal{P}_o^{r_1}$, is defined as [8],

$$\mathcal{P}_o^{r_1} = P \left[\left(\mathcal{R}_{r_1}^{d(I)} < \mathcal{R}_d^{th} \right) \cup \left(\mathcal{R}_{r_1}^{r_1} < \mathcal{R}_1^{th} \right) \right] \quad (38)$$

Putting (5) and (17) into above equation and setting $\Gamma_D = 2^{6\mathcal{R}_d^{th}} - 1$, $\Gamma_{R_1} = 2^{6\mathcal{R}_1^{th}} - 1$, we obtain

$$\mathcal{P}_o^{r_1} = \gamma \left(k_1, \frac{\max(u_1, u_2)}{\theta_1} \right) / \Gamma(k_1) \quad (39)$$

where $\gamma(\cdot, \cdot)$ is the lower incomplete Gamma function and

$$u_1 = \frac{\Gamma_D}{\rho_s[(\beta - \alpha\Gamma_D) + \sqrt{\beta^2(1 + \Gamma_D(1 - \kappa_1^2))}]}, \quad (40)$$

$$u_2 = \frac{\Gamma_{R_1}}{\rho_s[(\alpha - \lambda^2\beta\Gamma_{R_1}) + \sqrt{\alpha^2 + \alpha^2\Gamma_{R_1} + \lambda^4\beta^2\kappa_1^2\Gamma_{R_1}^2}]} \quad (41)$$

The expressions in (40) and (41) are obtained by substituting the corresponding achievable-rate expressions into (38), converting the logarithmic inequalities into SINR thresholds, and solving the resulting quadratic equations with respect to the channel gain g_{sr_1} . The physically meaningful positive roots are retained, yielding the thresholds u_1 and u_2 .

Now, the outage probability at R_2 , i.e. $\mathcal{P}_o^{r_2}$ is defined as [8],

$$\mathcal{P}_o^{r_2} = P \left[\left(\mathcal{R}_{r_2}^{d(II)} < \mathcal{R}_d^{th} \right) \cup \left(\mathcal{R}_{r_2}^{r_2} < \mathcal{R}_2^{th} \right) \right] \quad (42)$$

Using $\mathcal{R}_{r_2}^{d(II)L}$ from (20) above, we have $\mathcal{P}_o^{r_2} \leq \mathcal{P}_{oU}^{r_2}$, where

$$\mathcal{P}_{oU}^{r_2} = P \left[\left(\mathcal{R}_{r_2}^{d(II)L} < \mathcal{R}_d^{th} \right) \cup \left(\mathcal{R}_{r_2}^{r_2} < \mathcal{R}_2^{th} \right) \right] \quad (43)$$

Substituting $\mathcal{R}_{r_2}^{d(II)L}$ and $\mathcal{R}_{r_2}^{r_2}$ into (43) produces two quadratic inequalities in g_{sr_2} , given in (45) and (46). Their positive roots determine the minimum channel gains, g_{sr_2} , required to satisfy the target-rate requirements. Thus, we obtain

$$\mathcal{P}_{oU}^{r_2} = \mathbb{E}_{g_{r_1r_2}} \left[\gamma \left(k_2, \frac{\max(v_1, v_2)}{\theta_2} \right) / \Gamma(k_2) \right] \quad (44)$$

where, v_1 and v_2 are the positive roots of the quadratic equations (45) and (46), respectively, wherein $\mathcal{T} = G^2\Gamma_r + \lambda^4\rho_r^2g_{r_1r_2}^2\kappa_1^2\Gamma_r$ and $G = \lambda^2\beta\rho_s\kappa_2$ in (46).

B. Outage Probability Analysis at D

The outage probability at D , i.e. $\mathcal{P}_o^d$, is defined as [8],

$$\begin{aligned} \mathcal{P}_o^d &= P \left[\left(\mathcal{R}_{r_1}^{d(I)} < \mathcal{R}_d^{th} \right) \cup \left(\mathcal{R}_d^{(I)} < \mathcal{R}_d^{th} \right) \right] \\ &+ P \left[\left(\mathcal{R}_{r_2}^{d(II)} < \mathcal{R}_d^{th} \right) \cup \left(\mathcal{R}_d^{(II)} < \mathcal{R}_d^{th} \right) \right] \end{aligned} \quad (47)$$

Using $\mathcal{R}_{r_2}^{d(II)L}$ from (20) above, an upper bound on $\mathcal{P}_o^d$, denoted as $\mathcal{P}_{oU}^d$, can be obtained as

$$\begin{aligned} \mathcal{P}_o^d &\leq \mathcal{P}_{oU}^d = \mathbb{E}_{g_{r_1r_2}} \left[1 - \left(1 - \gamma \left(\frac{b_1}{\theta_1}, k_1 \right) \right) \times \right. \\ &\quad \left. \left(1 - \gamma \left(\frac{b_2}{\theta_4}, k_4 \right) \right) \left(1 - \gamma \left(\frac{b_3}{\theta_2}, k_2 \right) \right) \left(1 - \gamma \left(\frac{b_4}{\theta_5}, k_5 \right) \right) \right] \end{aligned} \quad (48)$$

The parameters b_1 , b_2 , b_3 , and b_4 represent the minimum channel-gain thresholds associated with the decoding conditions in the four involved links. These thresholds are obtained by solving the corresponding quadratic inequalities arising from the achievable-rate constraints. Here, b_1 is same as u_1 in (40) and b_2 , b_3 and b_4 are the positive roots of (49), (45) and (50), respectively.

$$P_r^2 g_{r_1d}^2 (1 - \kappa_1^2) + 2P_r g_{r_1d} - \Gamma_D < 0 \quad (49)$$

$$P_r^2 g_{r_2d}^2 (1 - \kappa_2^2) + 2P_r g_{r_2d} - \Gamma_D < 0 \quad (50)$$

To the best of our efforts, (44) and (48) cannot be obtained in closed-form; therefore, we obtain it using numerical integration.

It is worth noting that the outage analysis at R_1 admits a closed-form expression because the outage events depend on a single fading channel. In contrast, the outage probabilities at R_2 and D involve the inter-relay interference channel $g_{r_1r_2}$, resulting in coupled random variables and expectation terms. Therefore, closed-form solutions could not be obtained, and numerical integration is employed for evaluation.

IV. SIMULATION RESULTS

Simulation results are obtained using MATLAB, with all performance metrics averaged over 10^6 independent channel realizations. The source S and destination D are located at $(0, 0)$ and $(d, 0)$, respectively, where d denotes the reference S-D distance and is normalized to $d = 1$ without loss of generality. The relay users R_i , $i \in 1, 2$, are positioned at (x_{r_i}, y_{r_i}) . Unless otherwise stated, the parameters are set as [7], [8], [11]: $\alpha = 0.3$, $\sigma^2 = 1$, $\mathcal{R}_d^{th} = 0.2$, $\mathcal{R}_i^{th} = 0.2$, $m_{sr_i} = m_{r_i d} = m_{r_1 r_2} = 2$, $\lambda^2 = 0.2$, and $\lambda'^2 = 0.2$ for the with-IRI case. The average channel gain for a link of length d_{ij} is given by $\Omega_{ij} = (d/d_{ij})^\eta$, where the path-loss exponent is $\eta = 2$.

The two relay-user locations, L_1 and L_2 , are selected to represent two distinct and practically relevant relay-placement configurations. For L_1 , $(x_{r_1}, y_{r_1}) = (d/2, d/2)$ and $(x_{r_2}, y_{r_2}) = (d/2, -d/2)$, resulting in $d_{sr_1} = d_{sr_2} = d_{r_1 d} = d_{r_2 d} = d/\sqrt{2}$ and $d_{r_1 r_2} = d$. Hence, the corresponding average channel gains are $\Omega_{sr_1} = \Omega_{sr_2} = \Omega_{r_1 d} = \Omega_{r_2 d} = 2$ and $\Omega_{r_1 r_2} = 1$.

For L_2 , while maintaining $d_{r_1 r_2} = d$, R_1 is moved closer to S and R_2 closer to D , with coordinates $(x_{r_1}, y_{r_1}) = \left(\frac{d}{2} \left(1 - \frac{1}{\sqrt{2}} \right), \frac{d}{2\sqrt{2}} \right)$ and $(x_{r_2}, y_{r_2}) = \left(\frac{d}{2} \left(1 + \frac{1}{\sqrt{2}} \right), -\frac{d}{2\sqrt{2}} \right)$. These coordinates yield $d_{sr_1} = d_{r_2 d} = \frac{d}{2} \sqrt{2 - \sqrt{2}} \approx 0.3827d$ and $d_{r_1 d} = d_{sr_2} = \frac{d}{2} \sqrt{2 + \sqrt{2}} \approx 0.9239d$. Consequently, for

$$g_{sr_2}^2 \left[\rho_s^2 \left(1 - \beta^2 \kappa_1^2 - 2^{6R_D^{th}} \alpha^2 \right) \right] + g_{sr_2} \left[\rho_s^2 \lambda'^2 g_{r_1 r_2} \left(1 - \beta \kappa_1^2 - 2^{6R_D^{th}} \alpha \right) \right] + \left(1 - 2^{6R_D^{th}} \right) (1 - \kappa_1^2) \lambda'^4 \rho_r^2 g_{r_1 r_2}^2 < 0 \quad (45)$$

$$g_{sr_2}^2 \left(\rho_s^2 (2\lambda^2 \alpha \beta + \alpha^2) - \lambda^4 \beta^2 \rho_s^2 \Gamma_r (1 - \kappa_2^2) \right) + g_{sr_2} \left(2\alpha \rho_s G - 2\lambda^2 \beta \rho_s G \Gamma_r + \lambda^2 \lambda'^2 \beta \rho_s^2 \kappa_1 \kappa_2 g_{r_1 r_2} \right) - \mathcal{T} < 0 \quad (46)$$

$\eta = 2$, $\Omega_{sr_1} = \Omega_{r_2 d} \approx 6.828$, $\Omega_{r_1 d} = \Omega_{sr_2} \approx 1.172$, and $\Omega_{r_1 r_2} = 1$. Here, 'wo' and 'w' in the legends denote the cases without and with inter-relay interference (IRI), respectively. Performance is evaluated in terms of achievable rate and outage probability. For outage probability, both analytical (Ana) and simulation (Sim) results are shown for validation.

A. Achievable Rate Analysis

Figs. 1–3 illustrate the achievable rates of R_1 , R_2 , and D , respectively vs. SNR (dB) for both locations L_1 and L_2 .

In Fig. 1, for R_1 , identical achievable rates are observed for with-IRI and without-IRI scenarios at L_1 and L_2 . This behavior arises because R_1 decodes its message directly from the source without being affected by IRI. Regarding the comparison between IGS and PGS, at higher SNR regimes (above approximately 10 dB), IGS outperforms PGS by exploiting the asymmetric statistical properties of the transmitted signal, which allows it to better mitigate SIC in the system. Conversely, at lower SNR values, PGS marginally surpasses IGS. This is because at low SNR, the system is noise-limited rather than interference-limited, and in such a regime, the benefits of IGS's interference mitigation capability are diminished. Both L_1 and L_2 exhibit the same qualitative trend, confirming the consistency of the observations across different user locations.

In Fig. 2 for without-IRI scenario, the trend closely mirrors that of R_1 for both L_1 and L_2 . However, a markedly different behavior is observed in the with-IRI case. When IRI is present, PGS outperforms IGS across the mid-to-high SNR range. This can be attributed to the fact that R_2 receives interference from R_1 's transmission (IRI), which acts as an additional structured impairment. IGS may not optimally suppress this externally imposed IRI, particularly when the interference is strong. Nonetheless, it is noteworthy that below approximately -10 dB, IGS shows a tendency to again outperform PGS, suggesting that the advantage of IGS may re-emerge in extremely low-SNR regimes where the noise dominance suppresses the effect of IRI. Both L_1 and L_2 follow similar trends, with a performance gap between w-IGS and w-PGS at 30 dB on the order of approximately 15–20%.

Here in Fig. 3, both the without-IRI and with-IRI cases reveal a consistent trend: PGS outperforms IGS across the entire SNR range for both L_1 and L_2 . The destination D receives signals that have undergone two-hop relay processing, and the cumulative effect of interference accumulation at each hop means that the signal received at D under IGS contains residual improper components that are harder to decode.

In Figs. 2 and 3, the achievable rates at R_2 and D obtained without applying the inequality are also plotted and labeled as w-IGS-UB. As expected, the achievable rates derived using

the inequality lie below the corresponding w-IGS-UB curves throughout the considered SNR range, verifying that the analytical expressions indeed constitute valid upper bounds.

These results suggest that while IGS is beneficial for certain relay users, its advantages do not uniformly extend to all nodes in the VFD-NOMA network.

B. Outage Probability Analysis

Figs. 4–6 present the outage probability curves for R_1 , R_2 , and D , respectively, as a function of SNR (dB) for both locations L_1 and L_2 . The close agreement between analytical and simulation results across all figures validates the accuracy of the derived outage probability expressions.

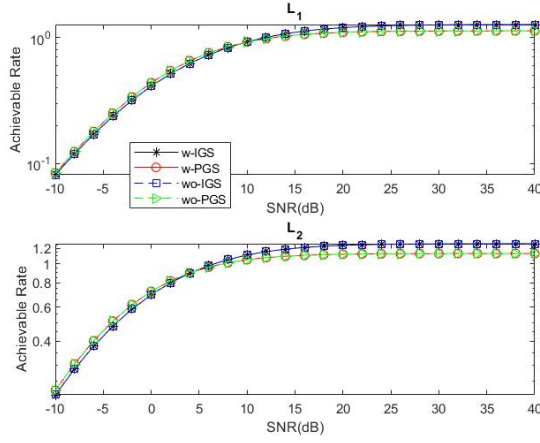
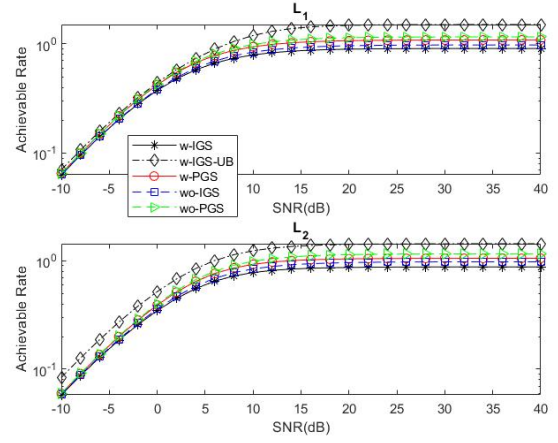
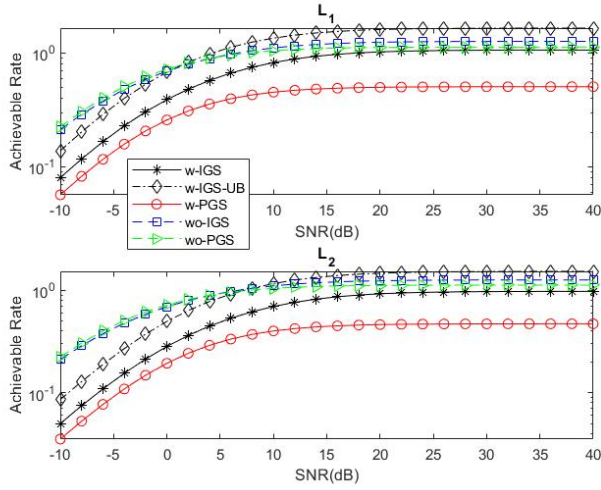
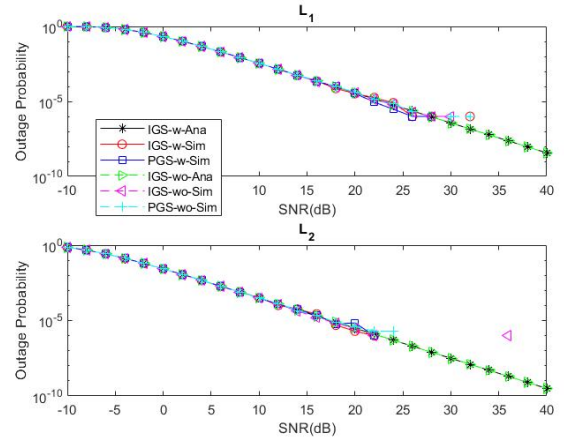
Consistent with the achievable rate observations, the outage probability curves in Fig. 4 for the without-IRI and with-IRI cases are nearly identical for R_1 in both L_1 and L_2 , further confirming that IRI has no influence on R_1 's performance. IGS and PGS exhibit very similar outage performance, with differences being negligible across the entire SNR range. This parity arises because at a relatively low threshold rate, both signaling schemes are capable of meeting the target rate requirement without a significant performance differential. Both L_1 and L_2 show the same behavior, with minor differences attributed to the distinct path loss and fading characteristics of each location.

In Fig. 5 without-IRI case, IGS achieves the same outage probability as PGS. However, in the with-IRI case, a more pronounced difference is observed: the IGS outage curves flatten out at a higher residual probability floor compared to PGS. This error floor behavior in IGS with IRI is a direct consequence of the irreducible interference caused by R_1 's transmission during phase-II that IGS cannot fully mitigate, resulting in the system entering an outage-limited regime regardless of SNR. L_2 follows a similar trend, though the absolute outage values differ slightly due to the different channel statistics.

As shown in Fig. 6, PGS achieves lower outage probability than IGS in the without-IRI scenario, while both schemes exhibit similar performance in the with-IRI case, the underlying reason has been discussed in the description of Fig. 3.

In Figs. 5 and 6, the outage probabilities at R_2 and D obtained without applying the inequality are additionally plotted and labeled as IGS-w-Sim-LB. As expected, the outage probabilities derived without using the inequality validate the derived lower-bound expressions.

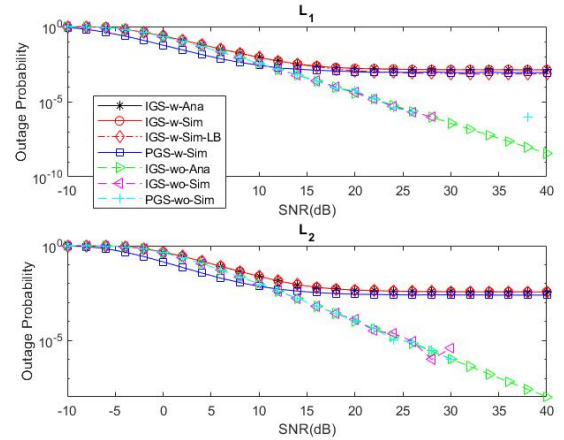
These results clearly indicate that PGS is the preferred scheme for the destination node in VFD-NOMA, even though IGS provides gains for specific relay users under certain conditions.


 Fig. 1: Achievable Rate vs. SNR at R_1 for L_1 and L_2 .

 Fig. 3: Achievable Rate vs. SNR at D for L_1 and L_2 .

 Fig. 2: Achievable Rate vs. SNR at R_2 for L_1 and L_2 .

 Fig. 4: Outage Probability vs. SNR at R_1 for L_1 and L_2 .

C. Effect of Threshold Rate Variation

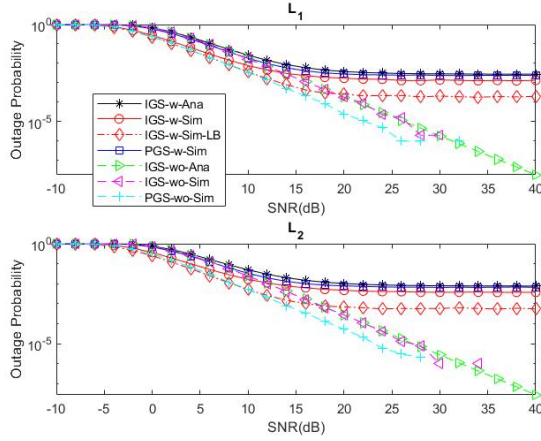
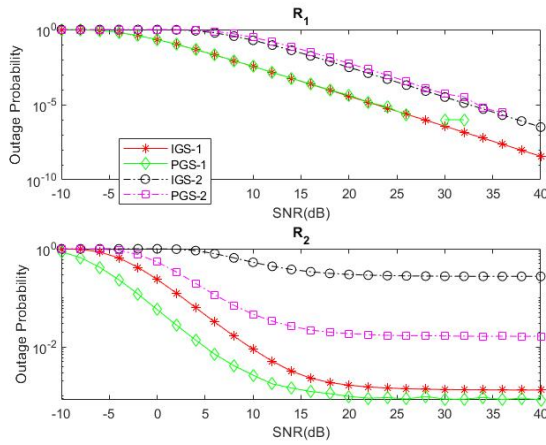
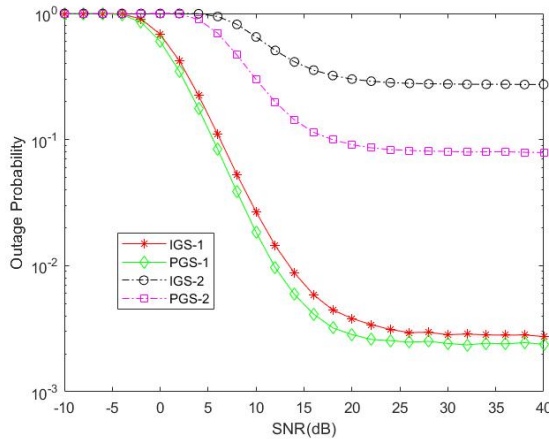
Figs. 7 and 8 illustrate the outage probability of R_1 , R_2 , and D at location L_1 with IRI, for two different threshold rates: $\mathcal{R}_i^{th} = \mathcal{R}_d^{th} = 0.1$ and $\mathcal{R}_i^{th} = \mathcal{R}_d^{th} = 0.4$. The legends IGS-1 and PGS-1 refer to the lower threshold, while IGS-2 and PGS-2 correspond to the higher threshold. As expected, a lower threshold rate results in significantly lower outage probabilities across all users and both signaling schemes, as a relaxed rate constraint is easier to satisfy even at moderate SNR.

For R_1 , IGS and PGS perform similarly at lower thresholds, with IGS offering marginal gains at higher thresholds. In contrast, PGS achieves lower outage probability for R_2 and consistently outperforms IGS at D , with a wider gap at higher thresholds. Overall, Figs. 7 and 8 show that IGS benefits certain relay nodes, whereas PGS is more robust at the destination, indicating that the optimal signaling scheme depends on the node role and interference conditions.


 Fig. 5: Outage Probability vs. SNR at R_2 for L_1 and L_2 .

V. CONCLUSION

This paper has presented a comprehensive performance analysis of IGS and PGS in a VFD-NOMA system with

Fig. 6: Outage Probability vs. SNR at D for L_1 and L_2 .Fig. 7: Outage Probability vs. SNR at R_1 and R_2 for L_1 .Fig. 8: Outage Probability vs. SNR at D for L_1 .

residual IRI and imperfect SIC over Nakagami- m fading channels. The results reveal that the relative merit of IGS over PGS is strongly dependent on the role of the node, the pre-

vailing interference conditions, and the operating SNR regime. Notably, IGS demonstrates increasing relative advantage for R_1 and R_2 at higher threshold rates, while PGS retains its superiority at D irrespective of the threshold.

Collectively, these findings motivate the design of node-selective hybrid signaling strategies for VFD-NOMA, where IGS is deployed at relay nodes operating in interference-limited, high-SNR conditions, and PGS is employed at the destination and at relay nodes subject to strong external IRI. Future work may explore the joint optimization of the IGS circularity coefficient and power allocation parameters to further maximize the performance gains in such heterogeneous signaling environments, as well as the extension of the proposed framework to multi-user VFD-NOMA systems and reconfigurable intelligent surface-assisted topologies.

REFERENCES

- [1] S. M. R. Islam, N. Avazov, O. A. Dobre, and K.-s. Kwak, "Power-domain non-orthogonal multiple access (noma) in 5g systems: Potentials and challenges," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 721–742, 2017.
- [2] A. Ahmed, X. Wang, A. Hawbani, W. Yuan, H. Tabassum, Y. Liu, M. U. F. Qaisar, Z. Ding, N. Al-Dhahir, A. Nallanathan, and D. W. K. Ng, "Unveiling the potential of noma: A journey to next-generation multiple access," *IEEE Commun. Surveys Tuts.*, vol. 27, no. 5, pp. 3099–3164, 2025.
- [3] Z. Ding, R. Schober, and H. V. Poor, "Unveiling the importance of sic in noma systems—part ii: New results and future directions," *IEEE Commun. Lett.*, vol. 24, no. 11, pp. 2378–2382, 2020.
- [4] D. Xu, "Improper gaussian signaling for uplink noma systems with finite blocklength codes under imperfect sic," *IEEE Transactions on Wireless Communications*, vol. 25, pp. 9923–9939, 2026.
- [5] H. Cheng, Y. Xia, Y. Huang, and L. Yang, "Improper gaussian signaling for downlink noma systems with imperfect successive interference cancellation," *IEEE Trans. Wireless Commun.*, vol. 21, no. 9, pp. 7753–7763, 2022.
- [6] T. Bhavsar and A. Jindal, "Cooperative noma aided improved connectivity in downlink through smart user pairing," in *2024 IEEE Pacific Rim Conference on Communications, Computers and Signal Processing (PACRIM)*, 2024, pp. 1–7.
- [7] —, "On reducing the outage probability in vfd-noma with limited csi at source," *IEEE Commun. Lett.*, vol. 27, no. 2, pp. 507–511, 2023.
- [8] J. Jose, P. Shaik, and V. Bhatia, "Vfd-noma under imperfect sic and residual inter-relay interference over generalized nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 25, no. 2, pp. 646–650, 2021.
- [9] H. Cheng, M. Zhang, M. Hua, Y. Xia, F. Ding, S. Zhang, W. Pei, and A. Lee Swindlehurst, "On the rate region of the downlink noma system with improper signaling and imperfect sic," *IEEE Transactions on Communications*, vol. 73, no. 9, pp. 8110–8124, 2025.
- [10] Y. Liu and H. Cheng, "Outage performance analysis and optimization of two-user downlink noma systems with imperfect sic and improper signaling," *IEEE Trans. Wireless Commun.*, vol. 23, no. 9, pp. 11 649–11 661, 2024.
- [11] Q. Y. Liao and C. Y. Leow, "Cooperative noma system with virtual full duplex user relaying," *IEEE Access*, vol. 7, pp. 2502–2511, 2019.
- [12] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channels; 2nd ed.* Newark, NJ: Wiley, 2005.
- [13] Y. Zeng, C. M. Yetis, E. Gunawan, Y. L. Guan, and R. Zhang, "Transmit optimization with improper gaussian signaling for interference channels," *IEEE Trans. Signal Process.*, vol. 61, no. 11, pp. 2899–2913, 2013.
- [14] I. Abu Mahady, E. Bedeer, S. Ikki, and H. Yanikomeroglu, "Sum-rate maximization of noma systems under imperfect successive interference cancellation," *IEEE Commun. Lett.*, vol. 23, no. 3, pp. 474–477, 2019.