

Localization of Directional Transmitters Using a Variance-Based Metric in Wireless Sensor Networks

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Abstract—Estimating the location of a transmitter is essential for applications such as regulatory enforcement, beacon tracking, and emergency response. This paper proposes the *variance-based metric* to identify the most likely transmitter location from a set of candidate positions using measurements collected by a sensor network. Unlike conventional approaches, the proposed method is effective with both directional and omnidirectional antennas. The proposed algorithm is evaluated against the weighted centroid and highest-received-power methods. For omnidirectional transmitters, the proposed algorithm performs almost as well as the weighted centroid, and better than the highest-received power method. However, for directional transmitters, the performance of weighted centroid localization degrades significantly, while the proposed metric consistently outperforms all evaluated techniques. These results demonstrate the proposed approach is superior in realistic deployment scenarios where some transmitters can be directional.

Index Terms—Transmitter localization, blind estimation position, sensor network, weighted centroid, variance-based metric.

I. INTRODUCTION

The localization of a transmitter using the received power measurements from multiple sensors is important for a variety of purposes [1]. In order to enforce regulations, spectrum regulators must detect and locate a transmitter operating at a location and frequency where transmissions are not allowed, or emitting power at levels above regulatory thresh-

olds [1]. For example, such capabilities are essential for keeping transmitters out of the exclusion zones within the Citizens Broadband Radio Service (CBRS) band [2] and the 6 GHz band [3]. Locating transmitters that violate regulation is especially critical for protecting passive services, such as radio astronomy, which are highly sensitive to interference [2], [4]. For example, the International Telecommunication Union has expressed grave concern regarding the growing use of transmitters that deliberately jam Global Position System (GPS) receivers [5]. Transmitter localization can locate such jammers. Other applications of localization include tracking targets that periodically transmit beacons, detecting intruders carrying wireless devices, and locating emergency or distress transmitters in support of public safety and rescue operations. A sensor network makes localization practical if algorithms can estimate transmitter location from sensor readings.

In this paper, we address the problem of estimating the location of a transmitter. We consider a network of N sensors, indexed by $s = 1, 2, \dots, N$. Each sensor s has a known location (x_s, y_s) and an omnidirectional antenna with gain G_s . At a given time, for each sensor s , received power P_s , is known. We consider the case where it is not possible to use angle of arrival localization techniques because the sensors do not estimate the direction of signal arrival [6], [7]. Similarly, time of arrival localization methods are not suitable, since sensors are not synchronized [8], [9], [10]. This work assumes that large empirical datasets required by machine learning localization methods are not available [1], [11], [12], [13], although established propagation models, such as Friis' equation or log-normal path loss [14], can be used. These assumptions are appropriate for many low-cost wide-area sensor networks. Moreover, we assume that the sensor designer does not have prior knowledge of the transmitter

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characteristics, such as the transmitter power and antenna pattern.

The most widely used localization solution for this scenario is the weighted centroid algorithm [7], [15], [16], [17], [6], [18], [19], [20],[21], which will be discussed in Section II. However, our previous work [21] shows why the weighted centroid performs poorly when a transmitter has a directional antenna.

In this paper, we propose a novel localization algorithm that operates under these assumptions and provides important advantages over approaches that are commonly used today, particularly when transmitters use directional antennas.

We summarize our contributions as follows:

- 1) We propose the *variance-based metric algorithm* to estimate the position of a transmitter. The algorithm selects the candidate position that results in the lowest variance of the radiated power estimates. We present the underlying assumptions, provide a detailed description of each step of the algorithm, and include all the relevant equations to implement the algorithm.
- 2) We provide guidance on how to select parameters for our algorithm that minimize the mean localization error, defined as the average distance between the estimated transmitter location and its true position.
- 3) We compare the effectiveness of our approach and other approaches and see how relative effectiveness depends on factors such as antenna pattern of the transmitter, sensor density, and variance of path loss.
- 4) We show that our algorithm achieves much lower localization error with a directional transmitters than the alternatives, and a slightly higher error than the best of the alternatives with an omnidirectional transmitter.

In Section II we present the weighted centroid method that we use for comparison. Section III describes our proposed variance-based localization technique. Section IV describes the simulation assumptions to compare the performance of all techniques. In Section V, we select the parameters for our proposed method. Section VI compares the performance of all techniques. Finally, VII presents our conclusions.

II. WEIGHTED CENTROID-BASED LOCALIZATION

Under the assumptions described in Section I, the weighted centroid approach is a widely used localization method [7], [15], [16], [17], [6], [18], [19], [20],[21]. This method can achieve a low localization error while having other benefits, such as low complexity and robustness. The transmitter location is estimated using the weighted centroid method [19], [16], [15] [18]:

$$(\hat{x}_t, \hat{y}_t) = \left(\frac{\sum_{s=1}^N w(s) \times x_s}{\sum_{s=1}^N w(s)}, \frac{\sum_{s=1}^N w(s) \times y_s}{\sum_{s=1}^N w(s)} \right) \quad (1)$$

where N is the number of sensors, $w(s)$ is the weight of the sensor s at position (x_s, y_s) and $(\hat{x}_t, \hat{y}_t)$ are the estimated coordinates of the transmitter.

In this paper, two established weighting approaches are considered: RSS-based weighting [19], [16], [15], [18], [20],

[21] and distance-based weighting [15], [21]. For each method, the equations for the weights are (2) and (3), respectively:

- RSS-based weighting:

$$w(s) = P_s^\rho \quad (2)$$

- Distance-based weighting:

$$w(s) = 10^{\rho \times P_s} \quad (3)$$

where ρ is a constant that belongs to the set of all positive real numbers and is set by the sensor network designer.

The authors of [15], [16], [18], [19] introduced a threshold parameter t . Sensors that have a received power less than t have $w(s) = 0$. This paper uses the same parameter t .

The other localization approach considered in this paper is to select the sensor with the highest received power and use its location as the estimated transmitter position. The weighted centroid approach is equivalent when $\rho \rightarrow \infty$.

As shown in [21], when transmitters are directional, both RSS-based and distance-based weighted centroid methods perform worse than simply using the location of the sensor with the highest received power, and the localization error with the weighted centroid grows as the transmitter power increases. Thus, the weighted centroid is probably a poor choice unless the sensor network designer is confident that all transmitters are omnidirectional. That paper [21] also showed the optimal ρ for distance-based weighted centroid varies greatly with transmitter characteristics such as transmitter power that the sensor network cannot know in typical scenarios, and a ρ that is optimal in one scenario may result in substantially higher localization error in another. This is a significant disadvantage of the approach.

III. VARIANCE-BASED METRIC

The proposed algorithm treats each candidate position (x_i, y_i) as a hypothetical transmitter location and evaluates how consistently it can explain the sensor measurements. This consistency is used as a measure of how likely the candidate is to correspond to the true transmitter location. If a candidate position is close to the true transmitter location, the estimated radiated power required to explain the observed received power measurements should remain approximately constant when evaluated at different distances along the same direction. Consistency is rewarded, whereas candidate positions that produce inconsistent radiated power estimates are penalized. The estimated transmitter location corresponds to the candidate position with the highest overall consistency.

To obtain a quantitative measure of how likely it is that a specific candidate position is the true location of the transmitter, we estimate the radiated power required to produce the observed received power along each angle θ . For a given angle, this estimation is performed at C predefined distances from the candidate position, referred to as reference distances. The resulting C radiated power estimates are then used to compute a variance. If the candidate position is close to the true transmitter location, the radiated power estimates should be similar across the reference distances along the same angle,

resulting in a small variance. Conversely, larger variances indicate that the candidate position is less consistent with the observed measurements. Therefore, candidate positions with smaller variances are considered more likely to correspond to the true transmitter location.

The proposed algorithm consists of four key steps. One step is to select the candidate positions for evaluation. The second step produces estimates of the received power at locations throughout the region using sensor measurements. The third step assigns a value to each candidate position that reflects how well a transmitter at that position could reproduce the estimated received powers. Finally, the last step selects the candidate position with the lowest assigned value as the estimated transmitter location. More specifically:

- 1) **Candidate Position Selection:** We first identify the sensor with the highest received power, denoted by (x_h, y_h) , as it is expected to be among the closest sensors to the transmitter. Candidate positions are then generated by placing a regular grid around this sensor, with one candidate at each grid intersection within a radius r of (x_h, y_h) . This localized search reduces computational complexity by restricting the search to a smaller region around the sensor with the highest received power.
- 2) **Estimation of the Received Power:** We assume that the received power is only known at sensor locations. However, the proposed algorithm requires received power estimates at the reference distances associated with each candidate position, where sensor measurements are generally not available. Therefore, we employ an interpolation function, $\hat{I}(x, y)$, to estimate the received power at any location (x, y) using the known received power measured by the sensors.
- 3) **Candidate Position Scoring:** To quantify how likely a specific candidate position (x_i, y_i) is to represent the true transmitter location, we examine the variance of the estimated radiated power values computed at C different predefined distances from the candidate position along a given angle θ . The set of predefined reference distances is denoted by $\{D_1, D_2, \dots, D_C\}$, where D_k represents the k -th reference distance. For any given angle θ , we compute the corresponding estimated radiated power $\hat{H}(x_i, y_i, \theta, D_k)$ emanating from the candidate position (x_i, y_i) that would produce the estimated received power $\hat{I}(x_i, y_i, \theta, D_k)$ that we observe at the point located at distance D_k for every reference distance D_k . Using the path-loss function $L(d)$ in dB that represents the expected path-loss as a function of distance d , we obtain

$$\hat{H}(x_i, y_i, \theta, D_k) = \hat{I}(x_i, y_i, \theta, D_k) \times 10^{E\left[\frac{L(D_k)}{10}\right]} \quad (4)$$

for all i from 1 to M ; for all k from 1 to C .

If the candidate position was the true transmitter location, the required radiated power for the same direction θ should be approximately the same for all distances. Therefore, a lower variance of the set of estimated

radiated powers $\{\hat{H}(x_i, y_i, \theta, D_k)\}_{k=1}^C$ calculated across different distances for a given θ indicates that (x_i, y_i) is more likely to correspond to the true transmitter location. The variance $\sigma^2(x_i, y_i, \theta)$ of the estimated radiated power for a candidate position (x_i, y_i) along a specific direction θ , is given by:

$$\sigma^2(x_i, y_i, \theta) = \frac{1}{C} \times \sum_{k=1}^C (\hat{H}(x_i, y_i, \theta, D_k) - \bar{h}(x_i, y_i, \theta))^2 \quad (5)$$

where

$$\bar{h}(x_i, y_i, \theta) = \frac{1}{C} \times \sum_{k=1}^C \hat{H}(x_i, y_i, \theta, D_k) \quad (6)$$

for all i from 1 to M ; for all k from 1 to C .

$\bar{h}$ is the mean of the estimated radiated power across $\{D_k\}_{k=1}^C$ for a transmitter located at (x_i, y_i) along the direction θ .

The variance of the estimated radiated power is expected to be small when the candidate position is close to the transmitter. However, a small variance may also occur when the candidate position is located far from any transmitter, resulting in a low estimated radiated power across the region. In the latter case, the mean estimated radiated power is also low, which makes these two scenarios indistinguishable based on variance alone. To address this ambiguity, we define a metric given by the variance of the estimated radiated power divided by its mean squared value.

To obtain an overall measure of consistency for each candidate position (x_i, y_i) , we evaluate the metric by integrating over the full 360° range. The variance-based metric is then defined as:

$$V(x_i, y_i) = \frac{\int_0^{360} \sigma^2(x_i, y_i, \theta) d\theta}{\int_0^{360} (\bar{h}(x_i, y_i, \theta))^2 d\theta} \quad (7)$$

for all i from 1 to M .

- 4) **Estimated Transmitter Location Selection:** The estimated transmitter location $(\hat{x}_t, \hat{y}_t)$ is the candidate location that minimizes the proposed variance-based metric $V(x_i, y_i)$:

$$(\hat{x}_t, \hat{y}_t) = \underset{(x_i, y_i)}{\operatorname{argmin}} V(x_i, y_i) \quad (8)$$

for all i from 1 to M .

Figure 1 illustrates a component of the proposed algorithm. The figure shows a candidate position (x_i, y_i) . For a given angle θ , three auxiliary points are defined at a distance D_k from the candidate position along that direction. At each of these points, the received power is estimated via interpolation. To support this estimation, a circular neighborhood of radius b is considered around each point, within which nearby sensor measurements are used. The green triangles represent the sensors contributing to the interpolation of the received power at these locations. Using the interpolated values at the three points, we compute the variance associated with the angle θ .

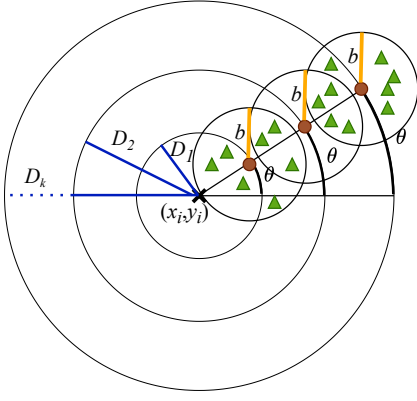


Fig. 1. Visualization of the proposed variance-based metric evaluated at a candidate position (x_i, y_i) for a given direction θ .

The same procedure is then repeated for all other angles under consideration.

In summary, the algorithm evaluates candidate positions by measuring the consistency of their inferred radiated power across distances and directions. The candidate with the lowest value of the variance-based metric $V(x_i, y_i)$ is chosen as the estimated transmitter location. This metric does not rely on exact signal values, making it adaptable to different propagation environments.

IV. SIMULATION ASSUMPTIONS

To evaluate the effectiveness of our approach as compared to the weighted centroid and the method based on the sensor with the highest received power, a series of simulations (using MATLAB tool) were performed.

The region simulated was large enough to produce the same results as an infinite plane. This implies that the region includes all relevant sensors, namely those that exceed the threshold $t = 5 \times 10^{-6}$ W in the weighted centroid method and those within the interpolation radius b for each candidate position with the variance-based metric. The density of candidate positions is 1 candidate position per square meter.

We evaluated all methods under two propagation models. For both models, the expected value of path loss for a given distance is known, although actual path loss may be random. In the first, path loss is deterministic line-of-sight propagation with path loss exponent $\gamma = 2$ as expected in free-space. In the second, we consider a non-line-of-sight path loss model that is random due to shadowing. Shadowing effects are modeled as independent log-normal random variables across sensors using the log-normal shadowing path loss model in (9) [14]:

$$L(d) = F(d_0) + 10\gamma \log\left(\frac{d}{d_0}\right) + \chi_g \quad (9)$$

where $L(d)$ is path loss in dB as a function of distance d . $F(d_0)$ is the path loss in dB at the reference distance $d_0 = 1$ m, calculated using the Friis model. χ_g is a Gaussian distributed random variable of zero-mean with standard deviation $\sigma = 3$ dB as in [22]. We use a path loss exponent, $\gamma = 2.5$, which

is typical of rural environments. The model is valid in the far-field region of the transmitting antenna, so distances are constrained to $d \geq d_0 = 1$ m [7]. Random path loss is more challenging for localization, as shadowing distorts the relationships between distance and received power.

We simulate three transmitter antennas: an omnidirectional antenna with gain function $G = 3$, and two directional antennas with gain function $G(\theta)$ from [23]:

$$G(\theta) = G_{\max} \left[e^{-\left(\frac{\theta}{\beta/2}\right)^2} + \alpha e^{-\left(\frac{\theta - \pi/2}{\beta/2}\right)^2} + \alpha e^{-\left(\frac{\theta + \pi/2}{\beta/2}\right)^2} \right], \quad (10)$$

where $G_{\max} = 3$ is the peak gain and $\alpha = 0.1$ determines the relative side-lobe level. The two directional antennas have beamwidths of $\beta = 30^\circ$ and $\beta = 15^\circ$, respectively. The antenna boresight is oriented at 45° . For a sensor located at (x_s, y_s) and a transmitter at (x_t, y_t) , the antenna gain toward the sensor is obtained by evaluating $G(\theta_s)$, where θ_s is the angle between the antenna boresight and the direction from the transmitter to the sensor. For both omnidirectional and directional antennas, transmitter power takes the values of 5, 50 and 100 W.

To implement the integral estimation in (7), we use a discrete approximation using a Riemann sum over angles 0 to 359° degrees with a step size of $\Delta\theta = 1$ degree. Let $\theta_j = j\Delta\theta$, where $j = 0, 1, \dots, 359$. The variance-based metric is then defined as:

$$V(x_i, y_i) = \frac{\sum_0^{359} \sigma^2(x_i, y_i, \theta_j)}{\sum_0^{359} (\hat{h}(x_i, y_i, \theta_j))^2} \quad (11)$$

Our approach requires some form of interpolation method to estimate the received power at locations where there are no sensors. Multiple methods have been proposed for doing so, such as Linear Interpolation [24], Spline Interpolation [25] and Kiring Interpolation [26]. For this paper, we used Inverse Distance Weighting (IDW) [27] [28], which works as follows:

If there is a sensor at location (x, y) , then estimated received power equals the received power measured by that sensor. Otherwise, received power is estimated as follows from the set $A(x, y, b)$ of all sensors within a radius b of (x, y) :

$$\hat{I}(x, y) = \frac{\sum_{i \in A(x, y, b)} \frac{P_i}{d_i^\phi}}{\sum_{i \in A(x, y, b)} \frac{1}{d_i^\phi}}, \quad \text{if } |A(x, y, b)| \neq 0 \quad (12)$$

$$\hat{I}(x, y) = 0 \quad \text{if } |A(x, y, b)| = 0 \quad (13)$$

where P_i is received power at sensor i , d_i is distance from location (x, y) to sensor i and ϕ is a constant that determines how weight changes with distance. (Typically $\phi = 2$) [29].

The radius of interpolation b is determined based on the sensor density such that the expected number of sensors within the interpolation region is $S=8$.

We compare the proposed method with the weighted centroid localization scheme, where the weighting parameter ρ controls the influence of the received signal measurements on the estimated transmitter position. In this work, we consider $\rho = 1$ for the RSS-based weighting as suggested by previous

research [16], [18], [19], [21] and $\rho = 0.07$ for the distance-based weighting as suggested by previous research [15].

Our technique is evaluated under two different assumptions regarding the received power. In the first scenario, a finite sensor density is assumed, and the received power is estimated through interpolation. In the second scenario, the received power is assumed to be known at every location. (Sensor locations should be irrelevant in this scenario, so when choosing candidate positions within radius r of the sensor with the highest received power at (x_h, y_h) we assume that the distance between (x_h, y_h) and the transmitter is negligible.)

V. PARAMETER SELECTION FOR VARIANCE-BASED METRIC

In this section, we select parameters for the variance-based metric (VBM) that can achieve low localization error. For simplicity, we assume reference distances are evenly spaced, so $D_k = a \times k$, where $k = 1, 2, 3, \dots, C$ and a is a constant. The following results guide the selection of parameters C and a . In all figures legends, 'VBM(Inter Pr)' and 'VBM(Actual Pr)' indicate the variance-based metric using interpolated received power and actual received power, respectively. 'Omni' and 'Dir' indicate omnidirectional or directional antennas, respectively. The directional antenna used in this section has a beamwidth of $\beta = 30^\circ$.

Figures 2 and 3 show mean localization error as a function of a for the deterministic and random path loss models, respectively. Sensor densities of 0.04, 0.06 and 0.1 sensors/m², and $S = 8$ are considered with interpolation. $C = 5$.

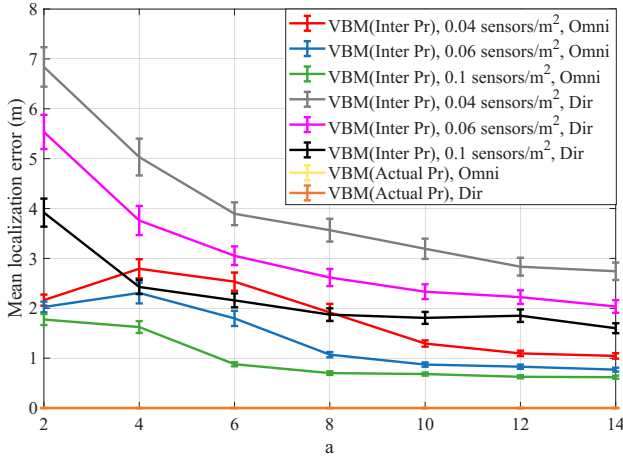


Fig. 2. Mean localization error as a function of a for the deterministic propagation model with $S = 8$ and $C = 5$. The directional antenna has $\beta = 30^\circ$.

Fig. 2 shows that with deterministic path loss, the mean localization error is always zero when actual received power is used. However, with interpolation, a should be as large as possible. This is because clusters of sensors that are far from the transmitter have less variation in received power. This reduces interpolation error, reducing mean localization error.

When the path loss is random, in Fig. 3, this advantage of a large a must be balanced with an advantage of a

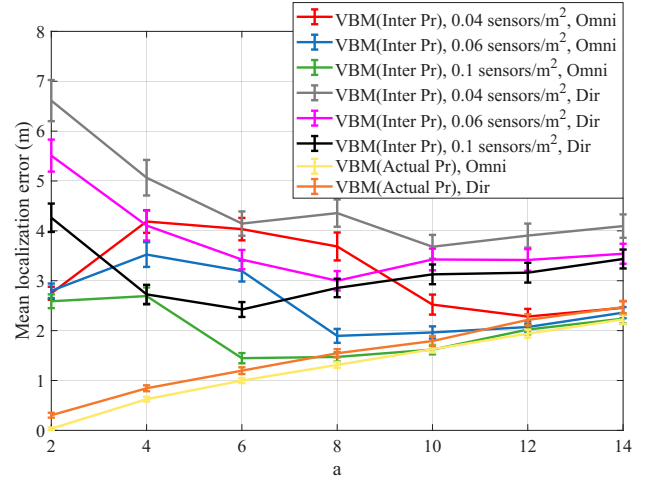


Fig. 3. Mean localization error as a function of a for the random path loss model with $S = 8$ and $C = 5$. The directional antenna has $\beta = 30^\circ$.

small a . This can be seen most clearly in the case where the received power is known everywhere. Here, the mean localization error increases with a because randomness in path loss introduces errors in the estimated received power, and these estimation errors have a greater impact when there is a lot of path loss, which is when a is large. For the case where we use interpolation to estimate received power, for a finite sensor density, both of the mechanisms described above are simultaneously present. If a is too small, interpolation errors dominate due to rapid power variation near the transmitter. In contrast, if a is too large, shadowing-induced errors in radiated power estimation dominate. Therefore, an intermediate value of a provides the best localization performance.

According to Fig. 3, for finite sensor densities, there exists an optimal a that minimizes the mean localization error. Moreover, the optimal value of a depends on sensor density: as sensor density increases, the optimal value of a decreases, whereas lower sensor densities require a larger value of a . This can be explained by the interpolation error. At lower sensor densities, the larger spacing between sensors results in greater interpolation errors, making higher values of a beneficial. In contrast, when the sensor density is high, interpolation errors are reduced and smaller values of a are sufficient to achieve good localization performance.

Based on the previous analysis, for deterministic path loss, this paper uses $a = 14$ for all sensor densities. For random path loss, we use $a = 10$ for a sensor density of 0.04 sensors/m², $a = 8$ for 0.06 sensors/m², and $a = 6$ for 0.1 sensors/m².

Figures 4 and 5 show mean localization error as a function of the number of reference distances, C , for the deterministic and random path loss models, respectively. In these figures, we show only the lowest and highest finite sensor densities, in addition to VBM with actual received power, using the corresponding values of a obtained from the previous analysis.

Increasing the number of reference distances C generally

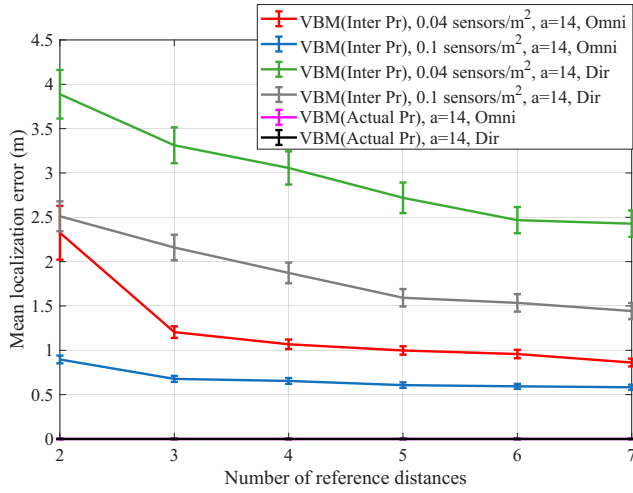


Fig. 4. Mean localization error as a function of number of reference distances for the deterministic propagation model with $S = 8$. The directional antenna has $\beta = 30^\circ$.

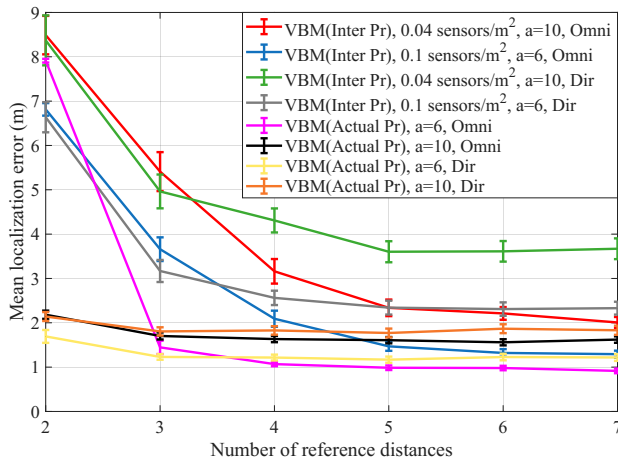


Fig. 5. Mean localization error as a function of number of reference distances for the random path loss model with $S = 8$. The directional antenna has $\beta = 30^\circ$.

reduces the mean localization error, as the algorithm can better capture variation in the estimated received power. However, this improvement exhibits diminishing returns: beyond $C=5$, further increases yield only marginal reduction in mean localization error, while the computational complexity continues to grow. Therefore, $C = 5$ is selected to achieve near-optimal performance while keeping computational effort low.

Although not shown here, we have found that the same parameter values also work well with the directional antenna with beamwidth $\beta = 15^\circ$.

VI. COMPARISON OF ALGORITHMS

This section compares the performance of the variance-based metric with the weighted centroid algorithm using both RSS-based and distance-based weights and with the algorithm that is based on the sensor with the highest received power.

In all figures, legend entries are labeled as “X, Y”, where X indicates the localization method used. This may correspond to the variance-based metric using either the actual or interpolated received power, the RSS-based weighted centroid with $\rho = 1$, the distance-based weighted centroid with $\rho = 0.07$, or the sensor with the highest received power method. The label Y specifies whether a deterministic ‘Det’ or random ‘Rand’ path-loss model is assumed. Solid lines correspond to deterministic path loss, dashed lines to random path loss.

Figures 6, 7 and 8 show mean localization error as a function of the transmitter power for three types of transmitter antenna, omnidirectional, directional with a beamwidth of $\beta = 30^\circ$, directional with a beamwidth of $\beta = 15^\circ$, respectively. Sensor density is 0.04 sensors/m^2 . We will subsequently show that results are similar with other sensor densities.

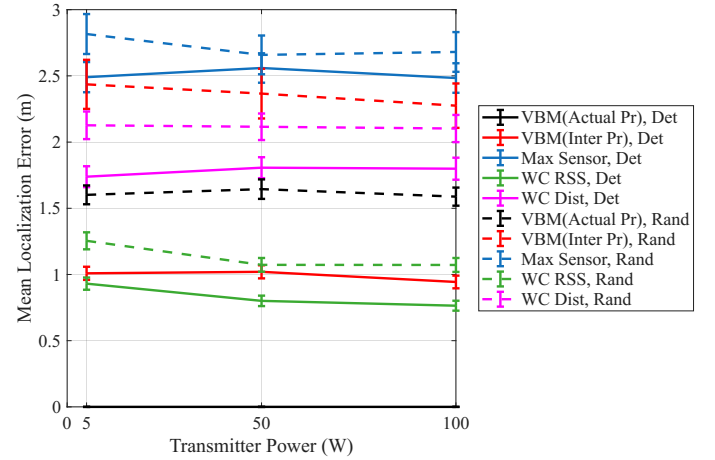


Fig. 6. Mean localization error as a function of transmitter power when transmitter antenna is omnidirectional, path loss is deterministic (solid lines) or random (dashed lines) and sensor density is 0.04 sensors/m^2 .

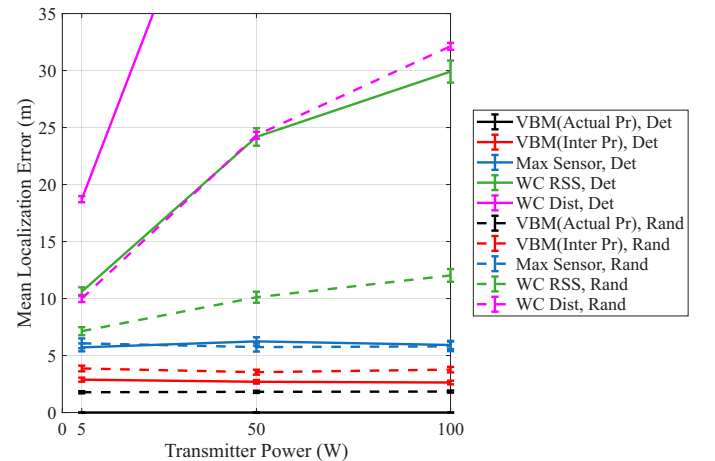


Fig. 7. Mean localization error as a function of transmitter power when transmitter antenna is directional with $\beta = 30^\circ$, path loss is deterministic (solid lines) or random (dashed lines) and sensor density is 0.04 sensors/m^2 .

These figures show that our algorithm consistently achieves a lower mean localization error than using the sensor location

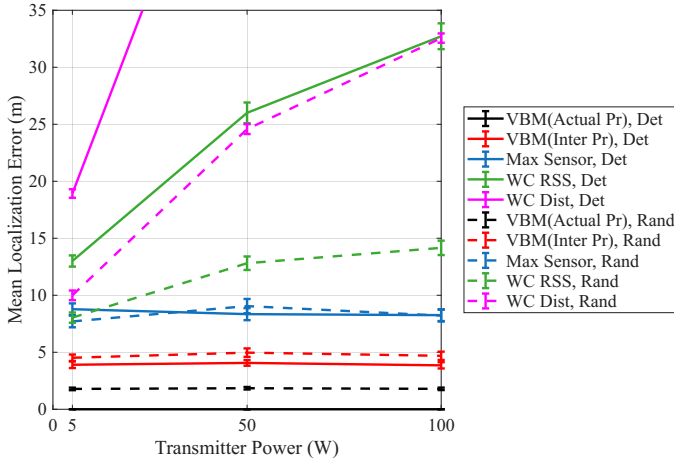


Fig. 8. Mean localization error as a function of transmitter power when transmitter antenna is directional with $\beta = 15^\circ$, path loss is deterministic (solid lines) or random (dashed lines) and sensor density is 0.04 sensors/m².

with the highest received power. This can be seen for all transmitter powers with both an omnidirectional transmitter (Fig. 6) and directional transmitters (Fig. 7 and 8), and with both deterministic path loss (solid lines) and random path loss (dashed lines). This is true when our algorithm estimates received power by interpolating sensor measurements (red lines), and our algorithm performs even better when it knows the actual received power at every point (black lines).

The relative performance of the variance-based metric and the weighted centroid depends greatly on characteristics of the transmitter and somewhat on other factors. When the transmitter is omnidirectional, as shown in Fig. 6, the localization error is comparable for the variance-based metric and the weighted centroid, and both perform better than using the location of the sensor with the highest received power regardless of transmitter power or path loss model. With deterministic path loss, localization error is lowest with the variance-based metric when the received power is known everywhere, followed by the RSS-based weighted centroid and then by the variance-based metric when the received power is estimated through interpolation. With random path loss, the RSS-based weighted centroid is better than the variance-based metric even when received power is known, and the difference is greater when received power is estimated. Overall, Fig. 6 shows that the RSS-based weighted centroid is the better choice when it is guaranteed that all transmitters are omnidirectional.

However, the variance-based metric with estimated received power (red lines) achieves much lower localization errors than all weighted centroid algorithms when antennas are directional, as shown in Fig. 7 and Fig. 8. The variance-based metric with the actual received power performs even better. Indeed, both the RSS-based and distance-based weighted centroids perform even worse than using the location of the sensor with the highest received power (blue lines). All of this is true for both deterministic (solid lines) and random path loss (dashed lines), and at all transmitter powers. Whereas the localization

error with the variance-based metric does not depend on the transmitter power, the localization error with the weighted centroid does, and error gets even worse as the transmitter power increases. When the antenna is directional, the weighted centroid chooses a point in the main lobe of the transmitter antenna, and as the transmitter power grows, both the RSS-based and distance-based weight centroid select a point further away from the transmitter in the direction of that main lobe as shown in [21].

Figure 9 shows mean localization error as a function of sensor density when the transmitter is omnidirectional. Increasing

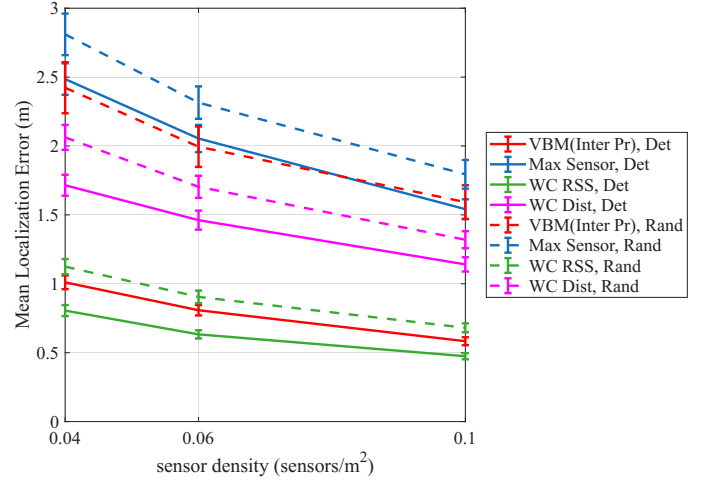


Fig. 9. Mean localization error as a function of sensor density with an omnidirectional transmitter. Path loss is both deterministic (solid lines) and random (dashed lines).

sensor density does not alter the relative performance of the considered algorithms. The variance-based metric, highest received power, and weighted centroid methods maintain the same ranking across all sensor densities. This also occurs with directional antennas, although these results are not presented due to space limitations.

VII. CONCLUSION

This paper proposed a new transmitter localization technique, referred to as a *variance-based metric*. The proposed metric evaluates how consistently each candidate position explains the measurements collected by the sensor network and uses this consistency as an indicator of the likelihood that the candidate corresponds to the true transmitter location.

The variance-based metric was compared with two widely used localization approaches: weighted centroid localization and the highest-received-power method. The weighted centroid algorithm was evaluated using both RSS-based and distance-based weighting functions. Performance was assessed through simulations considering omnidirectional and directional transmitter antennas, different sensor densities, transmitter power levels, and path-loss models.

We found that when all transmitters are omnidirectional, the variance-based metric performs reasonably well, although not quite as well as the weighted centroid approach with

RSS-based weighting. The RSS-based weighted centroid also outperformed the distance-based weighted centroid, and the highest-received-power method.

However, we see very different results with directional transmitters. The variance-based metric achieves much lower localization error than the highest-received-power method, and both the variance-based metric and highest-received power methods are vastly superior to both RSS-based and distance-based weighted centroid. The variance-based metric approach was superior across different sensor densities, transmitter powers, and path-loss models. Indeed, its advantage over weighted centroid localization increased with transmitter power.

Therefore, when there is a possibility that some of the transmitters being localized are directional, the variance-based metric is the best solution.

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