

Embedding Propagation Models into Neural Networks for Physics-Informed Localization

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Abstract—The need of precise and reliable real-time indoor localization is essential in applications like navigation for the visually impaired. Localization-based approaches have been moving more and more towards machine learning (ML) data-driven strategies instead of more traditional approaches like trilateration and triangulation. This is mostly due to the better generalization capabilities of ML-based methods, which, paired with fingerprinting datasets can provide a decent localization accuracy. However, ML-based methods are plagued by interpretability issues and remain as black-boxes. Their over-reliance on data in supervised methods often render them vulnerable to multi-path effects and non line-of-sight (NLOS) issues. Thus, in this work, we propose a physics-based localization approach by integrating propagation models into the training phase of a neural network (NN). Localization models are trained using varying influence of a physics loss to find a balance between real measured data and the theoretical propagation models behind it. Our physics-based approach improves model performance for certain positions in our custom dataset leading to a 9.80% overall increase in localization accuracy.

Index Terms—Physics-Informed Neural Network (PINN), Indoor Localization, Propagation Models.

I. INTRODUCTION

Precise localization in indoor and urban outdoor scenarios continue to be a challenging task especially towards the navigation of people with visual impairments. According to the World Health Organization (WHO), the number of visually impaired people is expected to triple by 2050 [1], [2]. Their integration in our society remains poor and their ability to navigate towards a desired destination in full autonomy is still a big challenge. Radio waves propagation in indoor scenarios, where Global Positioning System (GPS) is not very helpful, becomes much harder to predict using classical methods.

Traditional localization approaches use geometric techniques based on the Received Signal Strength Indicator (RSSI) data [3], [4]. The issue with these approaches is that the further we are from the transmitter, the more the fluctuations in RSSI contribute to the degradation of the localization performance. To avoid reliance on theoretical models, RSSI readings are measured at reference locations in a specific area in order to build a database called a radio map or a fingerprinting dataset [5], [6]. The measurements made during run-time are then compared with the measurements already in the database. This is done using either a simple nearest-neighbor matching algorithm or other advanced ML techniques. There are many

recent works that use fingerprinting with ML and Deep Learning (DL) [7]–[9]. To make these methods more robust, we tried in a previous work [10] to interpret and identify the impact of ML-based localization model key parameters which showed that the ML and DL models learn patterns from fingerprints but do not reveal which gateways contribute to the prediction and how multi-path, NLOS, or frequency-dependent propagation influence the result.

In this work, we aim to introduce propagation model parameters to ML-based localization methods. The idea is to derive a physics-based loss term to provide physical information to the DL model during the training phase in addition to ground truth data. The main idea of our work is as follows:

- We aim to move towards a direct PINN-based approach for localization taking into account the physical propagation models.
- We consider a logarithmic propagation model as it is easy to integrate into RSSI-based localization model, and build our own propagation model based on attenuation and shadowing parameters corresponding to our lab environment based on RSSI measurements collected previously.
- The propagation model is integrated into the localization model in order to derive a combined loss term based on data loss and physics loss.
- In order to balance information from data and the physical relationship between the RSSI and positions, an α regularization factor is determined to get the best localization accuracy overall.

The rest of the paper is organized as follows: Section II presents some related works. Section III presents the used dataset. Section IV presents the methodology and the used systems. In Section V, we go through results and discussions before concluding in Section VI.

II. EXISTING PINNS-BASED LOCALIZATION WORKS

While conventional DL approaches are mainly data-based, the mathematical relationship between different parts of the data and its inputs and outputs are also of interest. In [11], a fluid mechanics setting was used to introduce PINNs. Navier-Stokes equations, which relate the physics between spatial and velocity components of a fluid are used to influence the optimization of a NN. The velocity components predicted by

the NN are made to satisfy these equations while ensuring similarity to the ground truth data at same time. The resulting configuration allows for the use of more synthetic data as well since these equations will always be satisfied irrespective of the ground truth data. In the case of localization, such an approach has not been directly used albeit works involving physics-based learning approaches that could improve a downstream task like localization do exist. Such examples include propagation path-loss and RSSI prediction tasks. [12] use a DL-based approach for propagation path loss estimation in an urban environment in contrast to the usual model-based simulations like ray tracing and finite-difference time domain methods. [13] uses a PINN-based approach for path-loss prediction by deriving integral equations based on physical propagation models to use as a basis for the physics loss during the training phase of the NN. A similar approach for spectrum cartography is used in [14] where partial-derivative equations for RSSI are derived in terms of spatial coordinates for RSSI prediction in addition to the true RSSI values as ground truth. In [15], a PINN-based approach is used in a localization context to reduce the amount of training data needed by a seamless integration of the propagation model to generate synthetic data. While the aforementioned works were successful for their particular task, in this work we aim to further effectuate PINNs for enhancing localization accuracy.

III. MATERIALS AND DESCRIPTION OF THE DATABASE

We use an indoor fingerprinting dataset measured in the basement of the CEDRIC laboratory in CNAM Paris [16]. It consists of x and y coordinates mapped to RSSIs based on LoRa 868 MHz and 2.4 GHz. There are 6 gateways (GTWs) deployed on the floor. The dataset has 22,203 fingerprints collected at 63 coordinates. On average, there are 350 ± 60 fingerprints per coordinate in order to minimize the temporal RSSI fluctuations caused by shadowing and fading effects. The time delay between each fingerprint is 8 seconds. The area of the floor is 248 m². The floor plan is shown in Fig. 1. The green $\bullet$ symbols mark the location of coordinates where fingerprints were measured. The red $\otimes$ symbol marks inaccessible locations where no fingerprints were measured due to obstructions, resulting in poorly represented areas. The purple $\oplus$ symbol shows the specific GTW location including their (x, y) positions. The coordinates are normalized between 0 and 10 with (0,0) being the closest coordinate near the bottom left corner of the floor plan.

IV. SYSTEM MODEL

In this section, we introduce the proposed system model. Fig. 2 shows the overall pipeline of our work. Section IV-A explains the propagation parameter derivation process. Using these, the propagation model is then integrated with the general training algorithm explained in Section IV-B.

A. Derivation of Propagation Parameters

While fingerprinting datasets consist of real-time measured RSSI, there exist also the underlying propagation models that

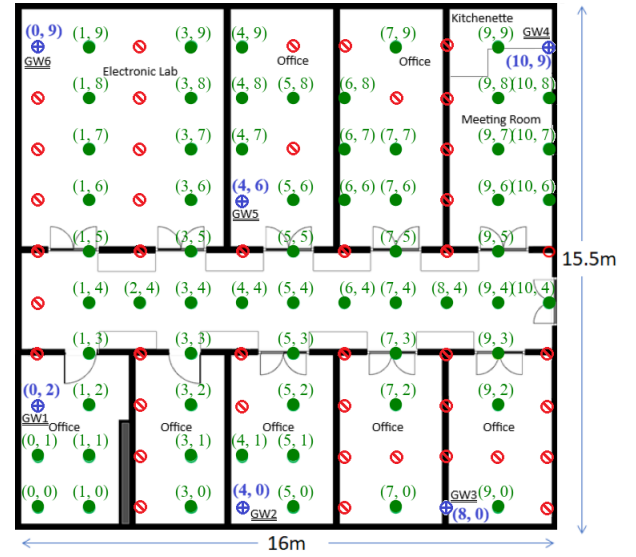


Fig. 1: Floor Map.

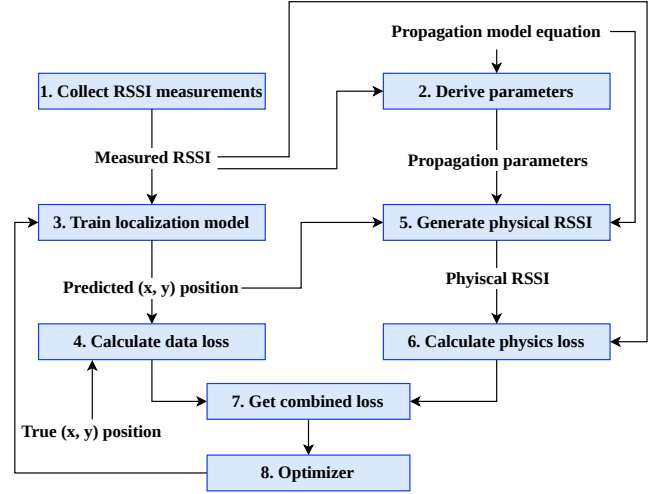


Fig. 2: Training pipeline incorporating a combined loss.

can mathematically describe the propagation of waves in a particular scenario. There are many standard propagation models catering to situations like indoor to outdoor propagation, propagation between floors and propagation through different materials [17]. Since this work is intended as a proof-of-concept regarding the impact of incorporating propagation-based physical constraints for localization and is not concerned with determining the most accurate propagation model for a specific environment, we rely on a simple and widely used logarithmic model that captures the distance-dependent attenuation and shadowing effects. Nonetheless, the proposed framework is not restricted to a specific model and can be extended to more sophisticated propagation models that better account for obstacles, floor penetration losses, and NLOS conditions. Our case general logarithmic propagation model

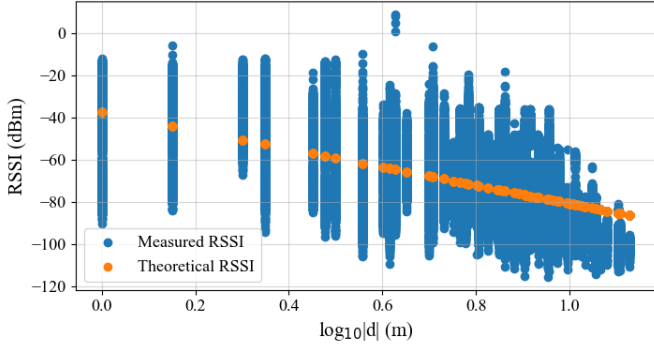


Fig. 3: Measured and theoretical RSSI in terms of distance between the position and the GTW on a logarithmic scale.

is given in the following equation:

$$RSSI = P_T - 10\eta \log_{10} |d| + \zeta(\sigma), \quad (1)$$

where P_T is the transmitted power, η is the path loss exponent, d is the distance between the transmitter and the receiver and $\zeta(\sigma)$ is a gaussian random variable presenting the shadowing effects. Henceforth, we will refer to 10η as a_1 , P_T as a_2 , RSSI as $\mathcal{F}$ and $\log_{10} |d|$ as $\mathcal{D}$; leaving us with the equation:

$$\mathcal{F} = a_2 - a_1 \mathcal{D} + \zeta(\sigma), \quad (2)$$

The distance $\mathcal{D}$ can be calculated using the distance between the physical coordinates and each gateway position as can be seen in the floor map provided in Fig. 1. We then solve the propagation model equation for a_1 and a_2 using the matrix equation $\mathbf{F} = \mathbf{X}\mathbf{D}$ where $\mathbf{F}$ and $\mathbf{D}$ are the matrix forms of $\mathcal{F}$ and $\mathcal{D}$, respectively, while $\mathbf{X}$ represents the unknowns a_1 and a_2 . Consequently, $\mathbf{X} = \mathbf{F}\mathbf{D}^{-1}$ can then easily be calculated using the Moore-Penrose inverse. Once we have the a_1 and a_2 values, we can plug them into the propagation model without the shadowing term to get theoretical RSSI fingerprints ($\mathcal{F}$):

$$\mathcal{F} = a_2 - a_1 \mathcal{D}, \quad (3)$$

The standard deviation of the difference of $\mathcal{F}$ and $\mathcal{F}$ gives us the σ we need for the shadowing $\zeta(\sigma)$ term. Fig. 3 shows the theoretical and measured RSSI distribution as a function of log-distance path loss for our LoRa 2.4 GHz fingerprints. The distribution of measured RSSI around the theoretical RSSI measures show the range of shadowing effects seen at each distance. The theoretical RSSI values do not trace through the center indicating the lack of enough measurements for a perfectly traced propagation model. However, it is of interest to pursue this approach as any improvement in localization here would correspond to even better localization when a more precise propagation model is possible. Alg. 1 gives the step by step process for the derivation of the propagation model parameters.

B. Methodology

The detailed pipeline is provided in Fig. 2. We use a regression-based localization using a general 1D CNN. The

Algorithm 1: DerivePropagationParameters($\mathcal{F}, \mathcal{C}, \mathcal{G}$)

Inputs : $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_n\}$, $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_n\}$, $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_m\}$ where $\mathcal{F}$ = RSSI Fingerprints, $\mathcal{C}$ = Coordinates, $\mathcal{G}$ = GTW Positions,

n = number of samples and m = number of GTWs

Outputs: $a_1 = 10\eta$, $a_2 = P_T$, σ = shadowing factor

Define :

$\mathcal{F}_{nm} = \{(\mathcal{F}_1^1, \mathcal{F}_1^2, \dots, \mathcal{F}_1^m), (\mathcal{F}_2^1, \mathcal{F}_2^2, \dots, \mathcal{F}_2^m), \dots, (\mathcal{F}_n^1, \mathcal{F}_n^2, \dots, \mathcal{F}_n^m)\}$;
 $\mathcal{D}_{nm} = \emptyset$;

1 Main

2 **foreach** $i = 1$ **to** n **do**

3 **foreach** $j = 1$ **to** m **do**

4 $\mathcal{D}_{nm} \leftarrow \log \sqrt{\mathcal{C}_i^2 - \mathcal{G}_j^2}$;

5 **end**

6 **end**

7 From (1): $RSSI = P_T - 10\eta \log |d| + \zeta(\sigma)$;

8 $\mathcal{F}_{nm} = a_2 - a_1 \mathcal{D}_{nm}$;

9 $\mathbf{F} = \mathbf{X}\mathbf{D} \Rightarrow \mathbf{F}\mathbf{D}^{-1} = \mathbf{X}$;

10

$$[\mathcal{F}_{nm}]_{1 \times nm} = [a_1 \quad a_2]_{1 \times 2} \begin{bmatrix} \mathcal{D}_{nm} \\ 1 \end{bmatrix}_{2 \times nm};$$

$$[a_1 \quad a_2]_{1 \times 2} = [\mathcal{F}_{nm}]_{1 \times nm} \begin{bmatrix} \mathcal{D}_{nm} \\ 1 \end{bmatrix}_{2 \times nm}^{\dagger};$$

where $[\cdot]^{\dagger} \Rightarrow$ Moore-Penrose pseudo-inverse;

11 Theoretical RSSI = $\mathcal{F}_{nm} = a_2 - a_1 \mathcal{D}_{nm}$;

12 $\sigma = std(\mathcal{F}_{nm} - \mathcal{F}_{nm})$;

13 **return** a_1, a_2, σ ;

network input is a single-channel vector of size 6 (one for each GTW). This only allows for two encoder layers before the latent space layer. This is followed by a dense layer before the output layer of size 2 (the x and y positions). Since the integration of physics is at the model optimization stage, any DL model can be used in the 3rd step of the flowchart seen in Fig 2. The idea is to show improvement on a general model so that the findings could be extended to a more complex model with even better localization accuracy.

The data used comprises the RSSI fingerprints for the floor map described in Section III. First, the propagation model parameters are determined using Alg. 1. The data is min-max normalized and training, validation and testing splits are created such that all fingerprints from one coordinate end up uniquely in one of the respective sets. This is done to ensure that all elements of the validation and testing data remain unseen to the model during training. The model is trained using the Mean Absolute Error (MAE) loss function with Adam optimizer. During the training phase, the model updates the weights based on both the distance loss and the physics loss. The distance loss ensures that the (x, y)

Algorithm 2: TrainModel($\mathcal{S}, \Theta, \mathcal{G}$)

Inputs : $\mathcal{S} = \{(\mathcal{F}_1, \mathcal{C}_1), (\mathcal{F}_2, \mathcal{C}_2), \dots, (\mathcal{F}_n, \mathcal{C}_n)\}$
 $\mathcal{S}_{train}, \mathcal{S}_{val}, \mathcal{S}_{test} = \text{SplittingData}(\mathcal{S})$
 $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_m\}$, model with weights Θ_0 ;
Outputs: Best model weights: Θ_{best}
Define :
Best Loss: $\mathcal{L}_{best}$, Loss Regularization: α ;
 $a_1, a_2, \sigma = \text{DefinePropagationParameters}(\mathcal{F}, \mathcal{C}, \mathcal{G})$;
 $\mathcal{D}(\hat{\mathcal{C}}, \mathcal{G}) = \text{foreach } j = 1 \text{ to } m \text{ do } \sqrt{\hat{\mathcal{C}}^2 - \mathcal{G}_j^2}$;
Hyperparameters:
epochs, batch_size, Learning Rate(Adam): lr ;
Loss Function: $\mathcal{L}_{fn} = \frac{1}{n} \sum_{i=1}^n |\hat{y}_i - y_i|$;

```
1 Main
2    $\mathcal{B}_{train} = \text{BatchLoader}(\mathcal{S}_{train}, \text{batch\_size})$ 
3   foreach epoch  $\in$  epochs do
4     foreach  $b \in \mathcal{B}_{train}$  do
5        $\hat{\mathcal{C}}_{train} = \text{model}(\mathcal{F}_{train})$ ;
6        $\mathcal{L}_{distance} = \mathcal{L}_{fn}(\hat{\mathcal{C}}_{train}, \mathcal{C}_{train})$ ;
7        $\hat{\mathcal{F}}_{train} = a_2 - a_1 \mathcal{D}(\hat{\mathcal{C}}_{train}, \mathcal{G}) + \zeta(\sigma)$ ;
8        $\mathcal{L}_{physics} = \mathcal{L}_{fn}(\hat{\mathcal{F}}_{train}, \mathcal{F}_{train})$ ;
9        $\mathcal{L}_{train} = \mathcal{L}_{distance} + \alpha \mathcal{L}_{physics}$ ;
10      UpdateWeights( $\mathcal{L}_{train}, lr, \Theta_b$ )
11    end
12     $\mathcal{L}_{val} = \text{Evaluate}(\text{model}, \mathcal{S}_{val})$ 
13    if  $\mathcal{L}_{val} < \mathcal{L}_{best}$  then
14       $\mathcal{L}_{best} = \mathcal{L}_{val}$ ;
15      SaveModel(model,  $\Theta_{best}$ )
16    end
17    model = LoadBestModel(model)
18     $\mathcal{S}_{test} = \text{Denormalize}(\mathcal{S}_{test})$ 
19     $\mathcal{L}_{test} = \text{Evaluate}(\text{model}, \mathcal{S}_{test})$ 
20  end
```

coordinates predicted by the model match the ones in the ground truth. The physics loss on the other hand, using the propagation model, ensures that these predicted coordinates are responsible for the same corresponding RSSI as seen in the ground truth. A regularization factor of α is used to control how much of the physics loss is added to the distance loss for the combined training loss seen during each epoch, with $\alpha = 0$ indicating only distance loss being used. The goal then is to find the right balance between distance and physics loss using hyperparameterization. Once the model is trained, the localization error is evaluated for the testing data using only the distance loss. Alg. 2 shows the overall training process.

V. RESULTS AND DISCUSSIONS

In this section, we evaluate the performance of the proposed framework at three levels of analysis. First, we assess its performance across different k-fold test sets. Next, we conduct a position-wise evaluation to analyze its behavior at each location using leave-one-out (LOO) cross validation. Finally, we

determine the optimal combination of loss function weights for each position to achieve the best localization accuracy.

A. Evaluation Across Multiple k-Fold Ensemble Settings

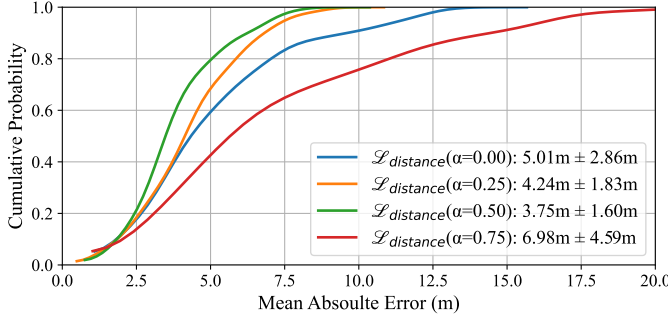
To obtain a more reliable estimate of the model's generalization and to demonstrate its effectiveness across different training configurations, the physics loss is integrated using various k-fold ensembles. For each fold, the model is trained on a subset of the data and evaluated on a distinct held-out subset. We evaluate multiple ensemble configurations although only two representative sets are presented in this paper. These results are sufficient to illustrate the general behavior, which remains consistent across all tested ensembles. Fig. 4 shows localization performance for two different k-fold scenarios while α is varied from 0 to 0.75. This enables us to ensure varying influence of physics loss on the training of our NN. Fig. 4a shows that using $\alpha = 0.5$ gives the best result for the training ensemble $k = 1$ while Fig. 4b shows that $\alpha = 0.75$ works the best for the fold $k = 2$. It must be noted here that these are both quite challenging ensembles, where the ratio of the training to testing data is low. We observe that the inclusion of the physics-based loss leads to improved localization performance. However, no common optimal value α is observed as it depends on the ensemble configuration.

B. Evaluation Using LOO Cross-Validation

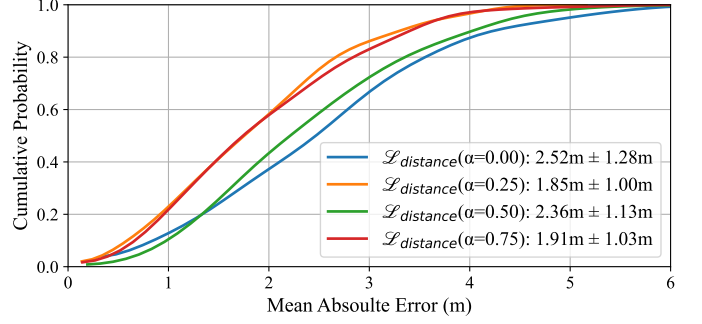
In this part, we expand our cross validation to a LOO scenario where the influence of physics loss during training is checked for every individual coordinate using different values of α . The main inspiration for this is to see if any propagation pattern can be determined specific to coordinates belonging to one room, being in the corner or next to a door.

1) *Position-Specific Analysis:* We analyze each location independently. Fig. 5 presents the localization error for coordinates (6, 4), (3, 6), (8, 4), and (9, 7) under different values of α . These examples are chosen to show the least localization error for the values of $\alpha = 0, 0.25, 0.5$ and 0.75 respectively. Here, we notice that different positions react differently to the amount of physics loss used during training. Therefore, α tuning should be performed individually for each location rather than applied globally.

2) *Evaluation Across the Entire Floor Plan:* We investigate whether positions located within the same room or sharing similar physical conditions (e.g., corners, behind doors, or near a GTW) exhibit similar behavior and can therefore use the same value of α . Fig. 6 shows the best localization error for each position, using LOO cross validation and the best value of α , on the floor map. The localization error on the floor map has been colored red, green, blue and purple for values of α as 0, 0.25, 0.5 and 0.75 respectively. We notice that there is no evident best value of α for the least localization error. Instead, some positions have low error with more influence of physics loss during training while others have higher error. While the initial idea of different error position seems to be verified, the difference is not as intuitive and is not observed to be associated with positions belonging to a specific room or with

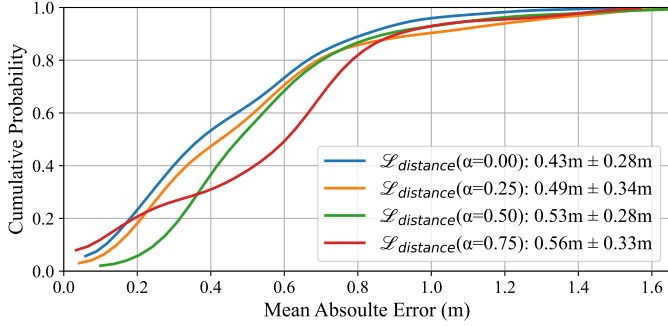


(a) CDF MAE with varying alpha for $k = 1$.

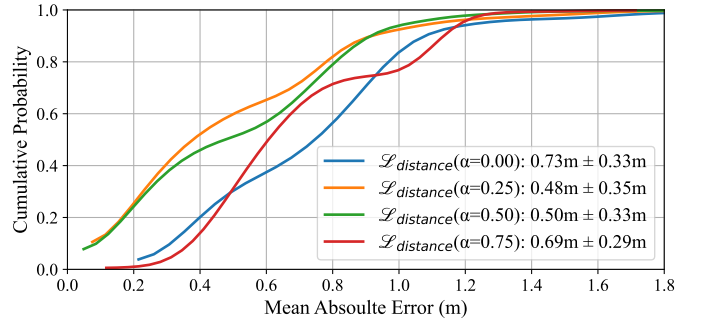


(b) CDF MAE with varying alpha for $k = 2$.

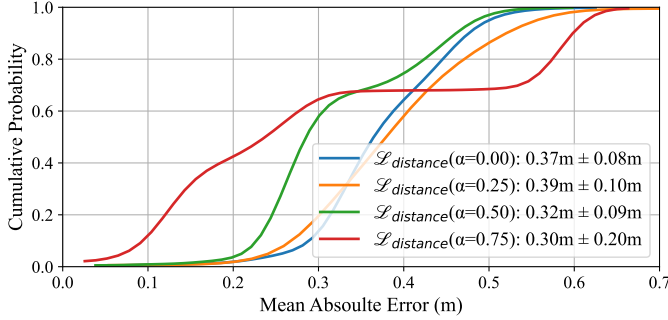
Fig. 4: Obtained Results using Multiple k-Fold Ensemble Settings.



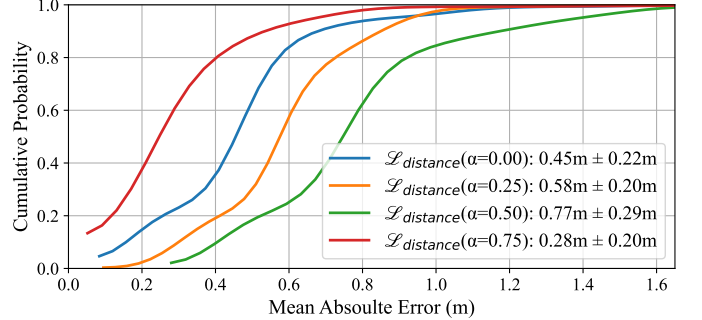
(a) LOO MAE CDF for (6, 4).



(b) LOO MAE CDF for (3, 6).



(c) LOO MAE CDF for (8, 4).



(d) LOO MAE CDF for (9, 7).

Fig. 5: CDF for some positions using different alpha values for physics loss.

TABLE I: MAE improvement when using physics loss after best model selection.

Method	MAE (m)	Relative Improvement (%)
Without α	1.34	—
Optimal α	1.21	9.80

a specific constraint. Nonetheless, a preliminary classification model can be used to determine the localization model to be used for each position. This way, we can benefit from the right amount of propagation model influence and minimize the localization error overall by selecting the best model per RSSI fingerprint.

3) *Selecting best model per position:* Table I shows localization error on the entire dataset using LOO cross validation

TABLE II: Mean, min and max localization error improvement for % of coordinates.

% Error Improved	% of coordinates		
	Mean Error	Min Error	Max Error
0-1%	3.17%	23.8%	22.2%
1-5%	27.0%	7.94%	28.6%
5-10%	22.2%	3.17%	17.5%
10-20%	28.6%	22.2%	22.2%
>20%	19.0%	42.9%	9.52%

using optimal α value and when no α is used (i.e only data loss). Overall, we record an improvement of 9.80% with the inclusion of the best models using physics loss in contrast with the use of data loss alone. To address the issue of selecting the appropriate value of α , a multiclass classification model

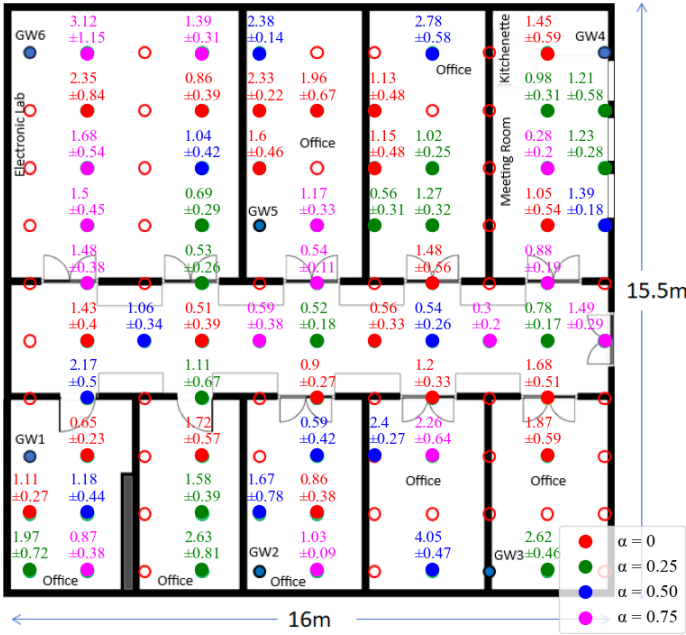


Fig. 6: Floor map with the best localization error for each position associated with the corresponding α value.

is trained offline such that each class corresponds to an α value associated with a specific localization model. During the online phase, the classifier predicts the most suitable α value based on the RSSI, and the corresponding localization model is then applied to estimate the user's position.

In Table II, for our 63 coordinates, we report the amount of localization error improvement for various ranges of error. The first column shows different ranges of error percentage improvement that has been achieved. The other columns show the percentage of coordinates for which the respective error improvement has been achieved. For example, the last row shows that 19% of the total coordinates reported a mean error improvement of more than 20%, 42.9% of the total coordinates reported a minimum error improvement of more than 20% and 9.52% of the total coordinates reported a maximum error improvement of more than 20%. In general, we notice that for each consecutive thirds of the coordinates, the percent increase in localization accuracy is up to 5%, 15% and over 20% respectively.

VI. CONCLUSION

In this work, we introduce a PINN-based approach for more accurate indoor localization. We derive physical propagation parameters for a LoRa 2.4 GHz dataset which are then integrated with a DL training phase to introduce a physics loss term in addition to the distance loss. This physics loss is determined by making sure that the positions predicted by the model during training are also physically associated with the same RSSI measurements. Our findings show that different positions on the floor map get affected differently by the inclusion and the amount of physics loss used during

training. However, a best model selection based on the RSSI ensures a 9.80% increase in localization accuracy overall in contrast with the use of distance loss only. Additionally, almost all the coordinates in the dataset see some level of percent increase in mean, min and max localization accuracy. This work can be further improved by adopting a more refined integration of the physics-based loss during training at the architectural level. Moreover, using a dedicated propagation model for each gateway, especially with larger datasets, could even further enhance the impact of the physics-informed loss on localization performance.

VII. ACKNOWLEDGMENT

The authors gratefully acknowledge the financial supports of the ANR program towards the HERBS project.

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