

Robust Huber-IRLS Channel Estimation for VLC Under Signal-Dependent and Impulsive Noise

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Abstract—This paper investigates channel-gain estimation in SISO visible light communication (VLC) systems impaired by thermal noise, signal-dependent shot noise (SDSN), and Bernoulli-Gaussian impulsive noise. A closed-form LS MSE expression is derived, the ML estimator is formulated from the Gaussian-mixture likelihood, and the CRLB is evaluated through numerical Fisher-information integration, with a component-separated approximation included for insight. To mitigate impulsive pilot samples without prior knowledge of the impulse probability or variance, an SDSN-normalized Huber-IRLS estimator is proposed. Simulations with fixed pilot amplitude show that LS approaches a non-zero high-SNR error level, while ML approaches the CRLB when full noise statistics are available. Huber-IRLS achieves near-ML performance with lower complexity and without requiring impulsive-noise parameters.

Index Terms—Visible light communication, channel estimation, signal-dependent shot noise, Bernoulli-Gaussian impulsive noise, Cramér-Rao lower bound, maximum-likelihood estimation, Huber-IRLS.

I. INTRODUCTION

Visible light communication (VLC) has attracted sustained interest as a complementary technology for indoor wireless access. By reusing existing lighting infrastructure and operating over an unlicensed optical spectrum, VLC can support data transmission where radio-frequency links are congested, restricted, or sensitive to electromagnetic interference [1]–[3]. Its relevance has further increased with heterogeneous sixth-generation (6G) indoor networks, where optical wireless links are expected to coexist with radio-frequency, millimeter-wave, and terahertz technologies [4]. In practical VLC receivers, information is conveyed through intensity modulation and recovered by direct detection (IM/DD), making the received electrical signal real-valued and dependent on the optical power collected by the photodetector [5].

Accurate channel estimation is a key requirement because the estimated gain directly affects subsequent detection and receiver-side processing. Unlike conventional radio-frequency models, VLC observations are not always well described by additive signal-independent Gaussian noise. Signal-dependent shot noise (SDSN) originates from photon-to-electron conversion, has a variance that scales with received optical intensity, and modifies the pilot likelihood, thereby affecting estimator structure and attainable accuracy [6]–[9]. Recent studies revisited VLC channel estimation under signal-dependent noise and

derived LS, ML, minimum mean-square-error, and Cramér-Rao-type benchmarks for deterministic and random channel models [7]–[9]. These works showed that ignoring the input-dependent variance creates a mismatch with the actual observation statistics, but mainly focus on thermal noise and signal-dependent optical noise.

Indoor optical receivers may also be exposed to sporadic high-amplitude disturbances produced by switching power supplies, fluorescent lamp ballasts, power-line interfaces, or pulsed electronic devices. Impulsive noise is often represented using Middleton Class-A, symmetric alpha-stable, or Bernoulli-Gaussian models [10], [11]. The Bernoulli-Gaussian model is convenient for estimation analysis because it separates background and impulsive states through a two-component Gaussian mixture [12]–[14]. Existing hybrid PLC-VLC studies mainly mitigate impulsive noise through blanking, clipping, or threshold processing in OFDM receivers [15]–[17], which is useful for demodulation but does not directly address pilot-aided VLC channel estimation under joint SDSN and Bernoulli-Gaussian impulsive contamination.

This limitation motivates the estimator considered in this work. When impulsive samples contaminate the pilot block, LS gives excessive influence to outlying observations, whereas exact ML accounts for the Gaussian-mixture likelihood but requires prior knowledge of the impulse probability and variance and involves nonlinear search. A practical estimator should therefore use the nominal VLC noise structure while limiting impulsive pilot samples without requiring Bernoulli-Gaussian parameters. The main contributions are threefold: i) a composite pilot-aided SISO VLC channel-estimation model is formulated under thermal noise, SDSN, and Bernoulli-Gaussian impulsive noise, with a closed-form LS MSE expression; ii) exact ML and Fisher-information-based CRLB expressions are developed from the resulting Gaussian-mixture likelihood, with a component-separated CRLB approximation included for insight; and iii) an SDSN-normalized Huber-IRLS estimator is proposed to reduce impulsive pilot samples without prior knowledge of the impulse probability or variance.

II. SYSTEM AND COMPOSITE NOISE MODEL

We consider a single-input single-output (SISO) VLC downlink composed of one LED transmitter and one photodiode

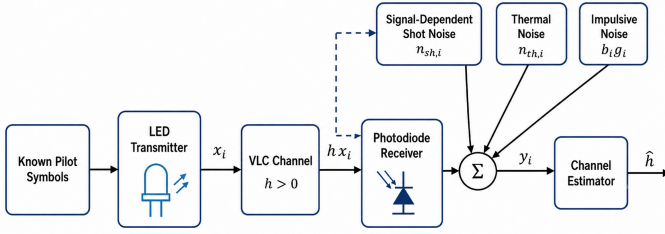


Fig. 1. Pilot-aided SISO VLC system model under thermal noise, signal-dependent shot noise, and Bernoulli-Gaussian impulsive noise.

receiver, as illustrated in Fig. 1. During the pilot phase, a block of N known positive pilot symbols $\{x_i\}_{i=1}^N$ is transmitted to estimate the deterministic effective channel gain $h > 0$. The gain h represents the scalar coefficient observed at the electrical receiver output and includes the optical propagation gain and receiver conversion factor [1]. The pilot symbols are assumed strictly positive, i.e., $x_i > 0$, which is consistent with IM/DD signaling and keeps the signal-dependent noise variance well defined.

After optical-to-electrical conversion and receiver noise addition, the i -th received pilot sample is modeled as

$$y_i = hx_i + \sqrt{hx_i} n_{sh,i} + n_{th,i} + b_i g_i, \quad (1)$$

where $n_{th,i} \sim \mathcal{N}(0, \sigma_{th}^2)$ is the thermal noise, while $n_{sh,i} \sim \mathcal{N}(0, \sigma_{sh}^2)$ is the normalized shot-noise component. The effective variance of the signal-dependent shot-noise term is

$$\text{Var}(\sqrt{hx_i} n_{sh,i}) = \sigma_{sh}^2 hx_i. \quad (2)$$

This representation captures the photon-counting nature of optical detection, where the shot-noise variance scales with the received signal level [6]–[8]. The impulsive-noise component is modeled as $b_i g_i$, where $b_i \sim \text{Bernoulli}(p_I)$ indicates the impulse occurrence and $g_i \sim \mathcal{N}(0, \sigma_I^2)$ denotes the impulse amplitude [11]. The random sequences $\{n_{th,i}\}$, $\{n_{sh,i}\}$, $\{b_i\}$, and $\{g_i\}$ are assumed mutually independent, with independent realizations across the pilot index. Since $b_i^2 = b_i$ and g_i is zero mean,

$$\text{Var}(b_i g_i) = \mathbb{E}[b_i^2] \mathbb{E}[g_i^2] = p_I \sigma_I^2. \quad (3)$$

Hence, the variance of the aggregate disturbance in (1) is

$$\sigma_{n,i}^2 = \sigma_{th}^2 + \sigma_{sh}^2 hx_i + p_I \sigma_I^2. \quad (4)$$

For a given impulsive-noise state, the received pilot sample follows a Gaussian distribution. When no impulse is present, i.e., when $b_i = 0$, the received pilot sample has mean hx_i and variance $\sigma_{0,i}^2$. This variance includes the thermal-noise variance and the signal-dependent shot-noise variance, and is given by

$$\sigma_{0,i}^2 = \sigma_{th}^2 + \sigma_{sh}^2 hx_i. \quad (5)$$

Thus,

$$y_i | h, b_i = 0 \sim \mathcal{N}(hx_i, \sigma_{0,i}^2). \quad (6)$$

When an impulse occurs, i.e., when $b_i = 1$, the received pilot sample remains Gaussian with the same mean hx_i , but its variance becomes larger. In this case, the new variance is $\sigma_{1,i}^2$, which is equal to $\sigma_{0,i}^2$ plus σ_I^2 . Therefore,

$$\sigma_{1,i}^2 = \sigma_{0,i}^2 + \sigma_I^2, \quad (7)$$

and

$$y_i | h, b_i = 1 \sim \mathcal{N}(hx_i, \sigma_{1,i}^2). \quad (8)$$

The additional term σ_I^2 represents the variance added by the impulsive noise.

Therefore, the conditional probability density function of y_i given h can be written as a Bernoulli-Gaussian mixture:

$$f(y_i; h) = (1 - p_I) \frac{1}{\sqrt{2\pi\sigma_{0,i}^2}} \exp\left(-\frac{(y_i - hx_i)^2}{2\sigma_{0,i}^2}\right) + p_I \frac{1}{\sqrt{2\pi\sigma_{1,i}^2}} \exp\left(-\frac{(y_i - hx_i)^2}{2\sigma_{1,i}^2}\right). \quad (9)$$

This distinction is important for the following analysis. The total variance is used in the LS MSE analysis. In contrast, the Bernoulli-Gaussian mixture density is used to formulate the exact ML estimator and to evaluate the CRLB based on the Fisher information. The proposed Huber-IRLS estimator uses only the nominal thermal-plus-SDSN variance and treats impulsive samples as outliers.

III. LEAST-SQUARES AND MAXIMUM-LIKELIHOOD CHANNEL ESTIMATION

This section presents two well-known estimators for the deterministic channel gain h under the observation model in (1). The LS estimator is first considered as a low-complexity reference, since it only exploits the linear pilot structure and does not require the full noise distribution. The ML estimator is then formulated from the Bernoulli-Gaussian mixture density in (9). The proposed Huber-IRLS estimator and the benchmark CRLB are developed in the following sections.

A. Least-Squares (LS) Estimator

The LS estimator provides a closed-form baseline for pilot-aided channel estimation. It is attractive from an implementation viewpoint because it does not require the probability density function of the noise and depends only on the observations received. Its main limitation is the use of a purely quadratic loss, which assigns a large contribution to high-amplitude residuals and therefore makes LS sensitive to impulsive pilot samples [7], [20]. For the received pilot sequence $\{y_i\}_{i=1}^N$, the LS problem is

$$\hat{h}_{LS} = \arg \min_h \sum_{i=1}^N (y_i - hx_i)^2. \quad (10)$$

Solving (10) yields

$$\hat{h}_{LS} = \frac{\sum_{i=1}^N x_i y_i}{\sum_{i=1}^N x_i^2}. \quad (11)$$

Although the physical VLC channel gain satisfies $h > 0$, (11) is kept as the conventional unconstrained LS benchmark. A positivity projection may be applied in practical implementations if required. In the following, we investigate the performance of the LS estimator by deriving its mean-square

error (MSE). Substituting (1) into (11) gives

$$\text{Error} = \hat{h}_{\text{LS}} - h = \frac{\sum_{i=1}^N x_i (\sqrt{h x_i} n_{\text{sh},i} + n_{\text{th},i} + b_i g_i)}{\sum_{i=1}^N x_i^2}. \quad (12)$$

For a fixed pilot sequence, all disturbance terms in (12) are zero mean and independent across pilot samples. Hence, under the adopted model,

$$\mathbb{E}[\hat{h}_{\text{LS}}] = h. \quad (13)$$

Using the second-order moment of the composite disturbance in (4), the LS mean-square error is obtained as

$$\begin{aligned} \text{MSE}_{\text{LS}} &= \mathbb{E}\{\text{Error}^2\} = \mathbb{E}\left[\left(\hat{h}_{\text{LS}} - h\right)^2\right] \\ &= \frac{\sum_{i=1}^N x_i^2 (\sigma_{\text{th}}^2 + \sigma_{\text{sh}}^2 h x_i + p_{\text{I}} \sigma_{\text{I}}^2)}{\left(\sum_{i=1}^N x_i^2\right)^2}. \end{aligned} \quad (14)$$

For a constant positive pilot block with $x_i = A_p$, where $A_p > 0$ denotes the pilot amplitude, (14) reduces to

$$\text{MSE}_{\text{LS}} = \frac{\sigma_{\text{th}}^2 + \sigma_{\text{sh}}^2 h A_p + p_{\text{I}} \sigma_{\text{I}}^2}{N A_p^2}. \quad (15)$$

This expression shows explicitly how the impulsive component enters the LS error through the product $p_{\text{I}} \sigma_{\text{I}}^2$. It can also be rewritten as

$$\text{MSE}_{\text{LS}} = \frac{\sigma_{\text{th}}^2}{N A_p^2} + \frac{\sigma_{\text{sh}}^2 h}{N A_p} + \frac{p_{\text{I}} \sigma_{\text{I}}^2}{N A_p^2}. \quad (16)$$

This form shows that increasing the pilot amplitude A_p reduces all three terms and drives the LS MSE toward zero. Therefore, the non-zero high-SNR level observed later is not a saturation with respect to A_p , but only a consequence of the fixed-amplitude simulation setting.

B. Maximum-Likelihood Estimator

The maximum-likelihood (ML) estimator uses the complete statistical description of the pilot observations. Unlike LS, which depends only on squared residuals, ML is built from the Bernoulli-Gaussian mixture density in (9). It is therefore considered as a statistical benchmark for a receiver that has access to p_{I} , σ_{I}^2 , σ_{th}^2 , and σ_{sh}^2 [20], [21]. Under the independence assumptions stated in Section II, the likelihood function is

$$\begin{aligned} L(h) &= \prod_{i=1}^N \left[(1 - p_{\text{I}}) \frac{1}{\sqrt{2\pi\sigma_{0,i}^2}} \exp\left(-\frac{(y_i - h x_i)^2}{2\sigma_{0,i}^2}\right) \right. \\ &\quad \left. + p_{\text{I}} \frac{1}{\sqrt{2\pi\sigma_{1,i}^2}} \exp\left(-\frac{(y_i - h x_i)^2}{2\sigma_{1,i}^2}\right) \right]. \end{aligned} \quad (17)$$

where $\sigma_{0,i}^2$ and $\sigma_{1,i}^2$ are defined in (5) and (7), respectively. The ML estimate is defined as

$$\hat{h}_{\text{ML}} = \arg \max_{h>0} \ln(L(h)). \quad (18)$$

The optimization in (18) does not yield a closed-form estimator. The gain h appears in the component means $h x_i$ and in the signal-dependent variances $\sigma_{0,i}^2$ and $\sigma_{1,i}^2$. In addition, the logarithm of a weighted sum of Gaussian densities prevents a direct analytical solution and may lead to a non-convex

objective. Since the VLC channel gain is positive, the numerical search can be restricted to the feasible domain $h > 0$. A one-dimensional search algorithm can be used for this purpose. Such algorithms are already implemented in many mathematical software tools.

In this paper, the ML estimator serves as a statistical benchmark under complete knowledge of the composite-noise parameters. Its dependence on p_{I} , σ_{I}^2 , σ_{th}^2 , and σ_{sh}^2 , together with the required nonlinear optimization, makes it less suitable for low-complexity and energy-constrained VLC receivers.

IV. PROPOSED HUBER-IRLS ESTIMATOR

Building on the limitations identified above, we propose an SDSN-normalized Huber M-estimator solved through iteratively reweighted least squares (IRLS) [18], [19]. The objective is to retain the nominal thermal-plus-SDSN structure of the VLC receiver while reducing the influence of impulsive pilot samples. The Bernoulli-Gaussian impulsive term is deliberately excluded from the residual normalization, since p_{I} and σ_{I}^2 are assumed unavailable at the receiver. Impulsive pilot samples are instead treated as outlying observations through Huber weighting. At iteration t , the normalized residual is defined as

$$r_i^{(t)} = \frac{y_i - \hat{h}^{(t)} x_i}{\hat{\sigma}_i^{(t)}}, \quad (19)$$

where the working variance is

$$\left(\hat{\sigma}_i^{(t)}\right)^2 = \sigma_{\text{th}}^2 + \sigma_{\text{sh}}^2 \hat{h}^{(t)} x_i. \quad (20)$$

This normalization compensates for the heteroscedastic SDSN term while preserving large normalized residuals caused by impulsive contamination. The Huber loss is

$$\rho_c(r) = \begin{cases} \frac{1}{2} r^2, & |r| \leq c, \\ c|r| - \frac{1}{2} c^2, & |r| > c, \end{cases} \quad (21)$$

where $c > 0$ controls the transition between quadratic and linear penalization. Small residuals are treated as in LS, while large residuals have bounded influence.

With $\hat{\sigma}_i^{(t)}$ fixed at iteration t , the corresponding IRLS weights are

$$w_i^{(t)} = \begin{cases} 1, & |r_i^{(t)}| \leq c, \\ \frac{c}{|r_i^{(t)}|}, & |r_i^{(t)}| > c. \end{cases} \quad (22)$$

For an impulsive pilot sample, $|r_i^{(t)}|$ tends to be large, which reduces $w_i^{(t)}$ and limits its contribution to the channel update. The next channel estimate is obtained from the weighted least-squares subproblem

$$\min_{h>0} \sum_{i=1}^N \frac{w_i^{(t)}}{\left(\hat{\sigma}_i^{(t)}\right)^2} (y_i - h x_i)^2. \quad (23)$$

The unconstrained solution of (23) is

$$\tilde{h}^{(t+1)} = \frac{\sum_{i=1}^N w_i^{(t)} x_i y_i / \left(\hat{\sigma}_i^{(t)}\right)^2}{\sum_{i=1}^N w_i^{(t)} x_i^2 / \left(\hat{\sigma}_i^{(t)}\right)^2}. \quad (24)$$

To preserve the physical positivity of the VLC channel gain

and the positivity of the SDSN working variance, the update is projected as

$$\hat{h}^{(t+1)} = \max\left(\tilde{h}^{(t+1)}, h_{\min}\right), \quad (25)$$

where $h_{\min} > 0$ is a small positive lower bound. Once $\hat{\sigma}_i^{(t)}$ and $w_i^{(t)}$ are fixed, the unconstrained weighted least-squares step has a unique solution whenever

$$\sum_{i=1}^N \frac{w_i^{(t)} x_i^2}{\left(\hat{\sigma}_i^{(t)}\right)^2} > 0. \quad (26)$$

The algorithm is initialized with

$$\hat{h}^{(0)} = \max\left(\hat{h}_{\text{LS}}, h_{\min}\right), \quad (27)$$

and stops when

$$\left|\hat{h}^{(t+1)} - \hat{h}^{(t)}\right| < \epsilon \quad (28)$$

or when a maximum number of iterations is reached. In the simulations, $c = 1.345$ is used as the standard Huber threshold for normalized residuals, providing high asymptotic efficiency under nominal Gaussian errors [18], [19]. This value is kept fixed in all experiments and is not tuned using p_1 or σ_1^2 .

V. CRAMÉR-RAO LOWER BOUND (CRLB)

The Cramér-Rao lower bound (CRLB) provides a lower bound on the variance of any unbiased estimator of the deterministic channel gain h . For an unbiased estimator $\hat{h}$, $\text{Var}(\hat{h}) \geq \frac{1}{\mathcal{J}(h)}$, where $\mathcal{J}(h)$ denotes the Fisher information associated with the pilot observation model [20], [21]. Following the Bernoulli-Gaussian mixture model in (9), the two weighted Gaussian components of the i -th pilot observation are written as

$$f_{0,i}(y_i; h) = (1 - p_1) \frac{1}{\sqrt{2\pi\sigma_{0,i}^2}} \exp\left(-\frac{(y_i - hx_i)^2}{2\sigma_{0,i}^2}\right), \quad (29)$$

$$f_{1,i}(y_i; h) = p_1 \frac{1}{\sqrt{2\pi\sigma_{1,i}^2}} \exp\left(-\frac{(y_i - hx_i)^2}{2\sigma_{1,i}^2}\right). \quad (30)$$

Thus, the corresponding mixture density is

$$f_i(y_i; h) = f_{0,i}(y_i; h) + f_{1,i}(y_i; h). \quad (31)$$

For the i -th pilot sample, the score function of the mixture is defined as

$$s_i(h; y_i) = \frac{\partial}{\partial h} \ln f_i(y_i; h) = \frac{\partial f_i(y_i; h)/\partial h}{f_i(y_i; h)}. \quad (32)$$

Using

$$\frac{\partial \sigma_{k,i}^2}{\partial h} = \sigma_{\text{sh}}^2 x_i, \quad k \in \{0, 1\}, \quad (33)$$

the score associated with the k -th Gaussian component is written as

$$s_{k,i}(h; y_i) = \frac{x_i(y_i - hx_i)}{\sigma_{k,i}^2} + \frac{\sigma_{\text{sh}}^2 x_i}{2} \left[\frac{(y_i - hx_i)^2}{\sigma_{k,i}^4} - \frac{1}{\sigma_{k,i}^2} \right], \quad (34)$$

where $k \in \{0, 1\}$. Since

$$\frac{\partial f_{k,i}(y_i; h)}{\partial h} = f_{k,i}(y_i; h) s_{k,i}(h; y_i), \quad k \in \{0, 1\}, \quad (35)$$

the mixture score in (32) can be written as

$$s_i(h; y_i) = \frac{f_{0,i}(y_i; h) s_{0,i}(h; y_i) + f_{1,i}(y_i; h) s_{1,i}(h; y_i)}{f_i(y_i; h)}. \quad (36)$$

Therefore, the Fisher information contribution of the i -th pilot sample is

$$\mathcal{J}_i(h) = \int_{-\infty}^{+\infty} s_i^2(h; y_i) f_i(y_i; h) dy_i. \quad (37)$$

Using (31) and (36), and since the pilot samples are independent, the exact Fisher information is

$$\mathcal{J}(h) = \sum_{i=1}^N \int_{-\infty}^{+\infty} \frac{[f_{0,i}(y_i; h) s_{0,i}(h; y_i) + f_{1,i}(y_i; h) s_{1,i}(h; y_i)]^2}{f_{0,i}(y_i; h) + f_{1,i}(y_i; h)} dy_i. \quad (38)$$

Accordingly, the exact CRLB is

$$\text{CRLB}_{\text{ex}}(h) = \frac{1}{\mathcal{J}(h)}. \quad (39)$$

The integral in (38) has no simple closed-form expression because the observation density is a Gaussian mixture whose mean and variances depend on h . It is therefore evaluated numerically.

A. Approximate Closed-Form CRLB

To obtain a simple analytical reference, we also consider a component-separated approximation of the Fisher information. The main idea is to treat the impulsive-noise state as if it were known. In other words, we first calculate the Fisher information for each Gaussian state separately, and then we average the two results using the Bernoulli probabilities. For the k -th Gaussian component, where $k = 0$ means no impulse and $k = 1$ means an impulse occurs, the received pilot sample has mean hx_i and variance

$$\sigma_{k,i}^2 = \sigma_{\text{th}}^2 + \sigma_{\text{sh}}^2 hx_i + k\sigma_1^2, \quad k \in \{0, 1\}. \quad (40)$$

Both the mean and the variance depend on the unknown channel gain h . Therefore, the Fisher information of the k -th Gaussian component is

$$\mathcal{J}_{k,i}(h) = \frac{x_i^2}{\sigma_{k,i}^2} + \frac{1}{2} \frac{(\sigma_{\text{sh}}^2 x_i)^2}{(\sigma_{k,i}^2)^2}, \quad k \in \{0, 1\}. \quad (41)$$

The first term comes from the dependence of the mean hx_i on h . The second term appears because the variance also depends on h through the signal-dependent shot-noise term. The component-separated Fisher information is then approximated by averaging the Fisher information of the two Gaussian states:

$$\mathcal{J}_{\text{app}}(h) = \sum_{i=1}^N [(1 - p_1) \mathcal{J}_{0,i}(h) + p_1 \mathcal{J}_{1,i}(h)]. \quad (42)$$

Therefore, the approximate closed-form CRLB is $\text{CRLB}_{\text{app}}(h) = \frac{1}{\mathcal{J}_{\text{app}}(h)}$. For a constant positive pilot block, where $x_i = A_p$ for all i , the approximate Fisher information becomes

$$\mathcal{J}_{\text{app}}(h) = N [(1 - p_1) \mathcal{J}_{0,A_p}(h) + p_1 \mathcal{J}_{1,A_p}(h)]. \quad (43)$$

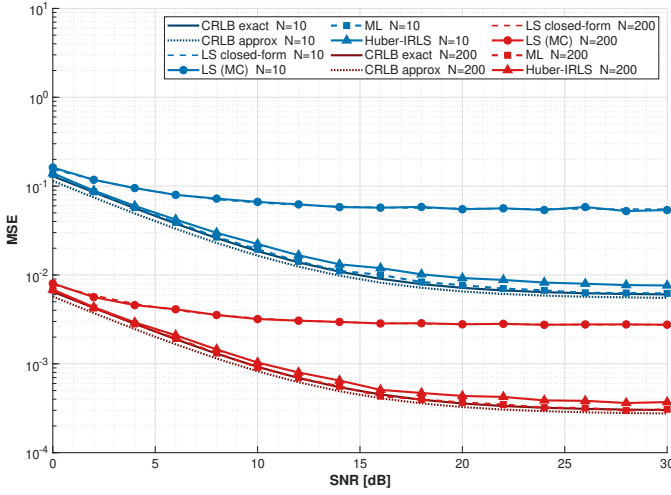


Fig. 2. MSE versus SNR for LS, ML, and Huber-IRLS with fixed pilot amplitude $A_p = 1$, $N \in \{10, 200\}$, $p_I = 0.1$, $\sigma_I^2 = 5$, and $\sigma_{sh}^2 = 0.05$.

Thus,

$$\text{CRLB}_{\text{app}}(h) = \frac{1}{N [(1 - p_I) \mathcal{J}_{0,A_p}(h) + p_I \mathcal{J}_{1,A_p}(h)]}. \quad (44)$$

Here,

$$\mathcal{J}_{k,A_p}(h) = \frac{A_p^2}{\sigma_{k,A_p}^2} + \frac{1}{2} \frac{(\sigma_{sh}^2 A_p)^2}{(\sigma_{k,A_p}^2)^2}, \quad k \in \{0, 1\}, \quad (45)$$

where

$$\sigma_{k,A_p}^2 = \sigma_{th}^2 + \sigma_{sh}^2 h A_p + k \sigma_I^2. \quad (46)$$

This approximation is not the exact Fisher information of the Bernoulli-Gaussian mixture. The exact Fisher information must be calculated from the full mixture density. Instead, this approximation treats the two Gaussian states separately and then averages their Fisher information values. Because this approximation uses extra information about the impulsive state, it can be optimistic compared with the exact mixture-based CRLB. Therefore, the exact CRLB remains the main theoretical benchmark, while the approximate closed-form CRLB is included only as a simple analytical reference.

VI. SIMULATION RESULTS AND DISCUSSION

All simulations are conducted in MATLAB with $h = 1$, $A_p = 1$, $A = 1$, $\sigma_{sh}^2 = 0.05$, and $N_{MC} = 5000$ Monte Carlo trials per point. Constant pilot symbols are set to $x_i = A_p$. The estimation SNR is defined as $\text{SNR} = h^2 A_p^2 / \sigma_{th}^2$. For the BER evaluation only, OOK data symbols are generated as $x_d \in \{0, 2A\}$, with $A = 1$. Unless otherwise stated, the default parameters are $p_I = 0.10$, $\sigma_I^2 = 5$, and $N = 50$. The Huber-IRLS threshold is fixed to $c = 1.345$, the standard value used for normalized Huber residuals, with $T_{\max} = 50$ and $\epsilon = 10^{-8}$. The positivity floor $h_{\min}$ is set to 10^{-6} .

The proposed Huber-IRLS estimator is compared with LS and with the ML estimator obtained by maximizing $\ln(L(h))$ in (18). The ML search is performed by grid search over $h \in [0.1, 3]$, which contains the true channel gain $h = 1$. The exact CRLB is evaluated numerically from (39) and used as the theoretical benchmark. For each operating point, the received

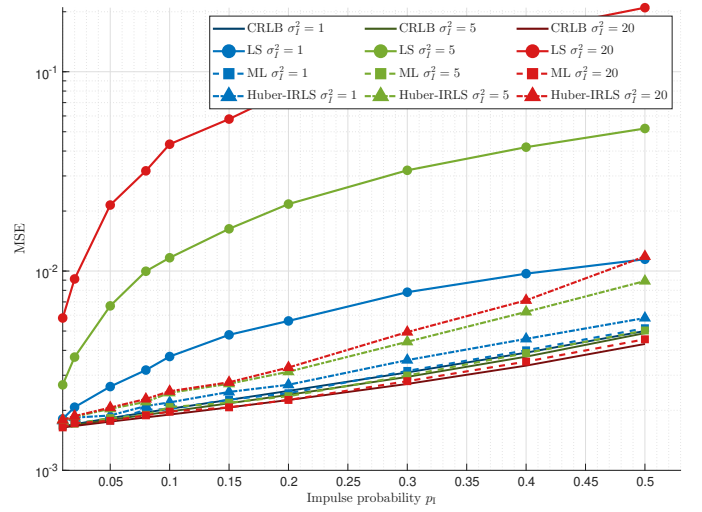


Fig. 3. MSE versus impulse probability p_I for different σ_I^2 , with $N = 50$, $\text{SNR} = 15$ dB, and $\sigma_{sh}^2 = 0.05$.

pilot samples are generated according to (1), and the empirical MSE is computed as $\text{MSE} = \frac{1}{N_{MC}} \sum_{\ell=1}^{N_{MC}} (\hat{h}^{(\ell)} - h)^2$.

A. MSE versus SNR

Fig. 2 compares the MSE of LS, ML, and Huber-IRLS for $N = 10$ and $N = 200$. The Monte Carlo LS curves follow the closed-form LS MSE, validating (14) and (15). In this figure, the pilot amplitude is fixed at $A_p = 1$, and the SNR is increased by decreasing σ_{th}^2 . In this fixed-amplitude setting, LS approaches a non-zero high-SNR error level because the SDSN and impulsive-noise terms remain after the thermal-noise contribution becomes small. As shown in (16), this behavior should not be interpreted as saturation with respect to A_p .

The ML estimator stays close to the exact CRLB since it uses the full Bernoulli-Gaussian likelihood and assumes complete noise-statistics knowledge. Huber-IRLS improves over LS and approaches ML in the pilot-rich regime, while avoiding prior knowledge of p_I and σ_I^2 . The approximate CRLB follows the exact CRLB closely in the tested regimes and is included only as an analytical reference.

B. MSE versus Impulse Probability

Fig. 3 studies the sensitivity of the estimators to the impulse probability p_I for $\sigma_I^2 \in \{1, 5, 20\}$, with $N = 50$ and $\text{SNR} = 15$ dB. As p_I increases, the LS estimator degrades rapidly, and the degradation becomes more pronounced for larger σ_I^2 . This behavior agrees with (14), where the average impulsive contribution enters the LS error through the product $p_I \sigma_I^2$.

The ML estimator remains close to the CRLB because it uses the full Bernoulli-Gaussian likelihood and assumes knowledge of the impulsive-noise parameters. The proposed Huber-IRLS estimator shows a slower MSE increase than LS, especially for large σ_I^2 . This is consistent with the weighting rule in (22): high-amplitude impulsive samples produce large normalized residuals and are assigned smaller weights in the channel update.

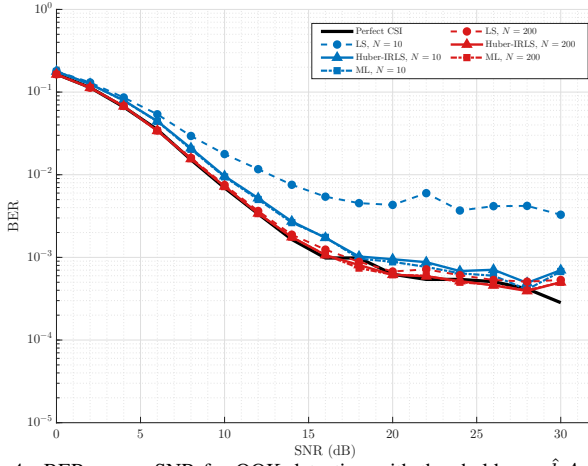


Fig. 4. BER versus SNR for OOK detection with threshold $\tau = \hat{h}A$, under $p_I = 0.1$, $\sigma_I^2 = 5$, and $N \in \{10, 200\}$.

At larger p_I , the gap between robust and non-robust estimation becomes more visible. The approximate CRLB may depart from the exact CRLB under stronger impulsive contamination, since the component-separated approximation does not fully capture the interaction between the two Gaussian mixture states. The exact CRLB is therefore kept as the main theoretical benchmark.

C. BER Performance Analysis

Fig. 4 reports the BER of OOK detection using the threshold $\tau = \hat{h}A$. The impulsive component is applied during pilot transmission, so the BER degradation reflects channel-estimation errors rather than direct data corruption. The perfect-CSI curve gives the ideal reference. For $N = 10$, the LS-aided receiver exhibits a visible high-SNR BER floor under the fixed-amplitude simulation setting, consistent with the MSE behavior in Fig. 2. Huber-IRLS reduces this floor by limiting the influence of impulsive pilot samples. For $N = 200$, both Huber-IRLS and ML move close to the perfect-CSI curve, confirming that a longer pilot block improves the reliability of the OOK decision threshold.

D. Computational Complexity

LS has the lowest cost, $\mathcal{O}(N)$, but is not robust to impulsive pilot samples. Exact ML evaluates the Gaussian-mixture likelihood over K candidate channel gains, leading to $\mathcal{O}(NK)$ complexity and requiring p_I , σ_I^2 , σ_{th}^2 , and σ_{sh}^2 . In contrast, Huber-IRLS has complexity $\mathcal{O}(NT)$, where T is the number of IRLS iterations. It avoids the impulsive-noise parameters and uses only the nominal thermal-plus-SDSN variance, providing a lower-complexity robust alternative to grid-based ML.

VII. CONCLUSION

This paper studied channel-gain estimation for SISO VLC systems under thermal noise, SDSN, and Bernoulli-Gaussian impulsive noise. We derived the LS MSE, exact ML estimator, and Fisher-information-based CRLB, and proposed an SDSN-normalized Huber-IRLS estimator that downweights impulsive pilots without requiring p_I or σ_I^2 . Simulations with fixed pilot amplitude showed that LS approaches a non-zero high-SNR

error level, ML approaches the CRLB when full noise statistics are known, and Huber-IRLS achieves near-ML performance with lower complexity. Future work will consider relative intensity noise, data-phase impulsive disturbances, and MIMO-VLC systems.

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