

Neural-Residual LMMSE Estimation for Measured V-VLC Channels Under Composite Optical Noise

Asma Bouhlel*, Imen Miled*, Salama Ikki[†], and Anis Sakly*

*LASEE Laboratory, ENIM, University of Monastir, Monastir, Tunisia

Emails: bouhlel_asmaa@yahoo.fr, imen.miled88@gmail.com,
sakly_anis@yahoo.fr

[†]Department of Electrical and Computer Engineering, Lakehead University, Thunder Bay, ON, Canada

Email: sikki@lakeheadu.ca

Abstract—Channel estimation in vehicular visible light communication (V-VLC) is challenging because the measured channel frequency response (CFR) depends on inter-vehicle distance, receiver inclination, ambient illumination, lane condition, LED configuration, and optical noise. This paper addresses pilot-aided reconstruction of measured V-VLC CFR vectors under thermal noise, signal-dependent shot noise (SDSN), and Bernoulli-Gaussian impulsive disturbances. Unlike data-driven V-VLC channel-modelling approaches that infer the CFR mainly from physical descriptors, and unlike VLC estimators focused on scalar channel-gain recovery, the proposed method estimates the full measured CFR vector from noisy pilot observations. An empirical linear minimum mean square error (LMMSE) estimator first exploits cross-frequency CFR statistics. A residual multilayer perceptron then refines the LMMSE estimate using pilot-block features, V-VLC contextual information, and the operating SNR. The resulting estimator, referred to as Neural-Residual LMMSE (NR-LMMSE), is evaluated on a measured V-VLC CFR dataset comprising 1345 measured channel conditions and 19 electrical modulation-frequency points from 200 kHz to 2 MHz. LS, empirical LMMSE, bias-corrected LMMSE, LS-based neural residual estimation, and context-only neural prediction are considered as baselines and ablation models. Results show that NR-LMMSE provides limited additional gain under AWGN and AWGN+SDSN, where empirical LMMSE is already well matched, but improves dB-domain CFR reconstruction and sample-wise robustness under composite signal-dependent and impulsive noise. Under composite noise, the mean absolute error decreases from 0.858 dB for empirical LMMSE to 0.296 dB, while the 95th-percentile sample-wise NMSE decreases from 1.59×10^{-1} to 1.11×10^{-2} .

Index Terms—Vehicular visible light communication, channel frequency response, channel estimation, LMMSE, neural residual learning, signal-dependent shot noise, impulsive noise.

I. INTRODUCTION

Visible light communication (VLC) is an optical wireless technology that uses light-emitting diodes (LEDs) for illumination and data transmission. By exploiting the visible optical spectrum, VLC can complement radio-frequency (RF) systems in scenarios affected by spectrum congestion, electromagnetic interference, or security constraints. These properties have motivated VLC research for indoor access, localization, intelligent transportation systems, and hybrid RF/optical networks [1]–[3]. Vehicular visible light communication (V-VLC) is one of the most relevant outdoor VLC applications. In V-VLC systems, vehicle headlamps, taillights, traffic lights, and road-side

lighting units can act as optical transmitters, while photodiodes or imaging sensors are used as receivers. Compared with RF vehicular links, V-VLC provides directional propagation, a reduced interference footprint, license-free operation, and immunity to RF electromagnetic interference. However, practical V-VLC links remain sensitive to line-of-sight availability, vehicle mobility, receiver inclination, ambient illumination, lane condition, LED configuration, weather, and road reflections [4]–[7].

These factors make the V-VLC channel strongly dependent on both geometry and environment. A scalar channel-gain model does not capture the frequency-dependent behavior of the optical link. A measured channel frequency response (CFR) provides a richer representation because it captures channel variation over frequency. Measurement-based V-VLC studies have therefore used physical descriptors, such as inter-vehicle distance, ambient light, receiver inclination, lane condition, and LED type, to model path loss and CFR from road measurements [8]–[10]. These works are important for channel characterization and data-driven channel modeling. However, they do not address receiver-side pilot-aided reconstruction of measured CFR vectors from noisy observations. Accurate channel estimation is essential for reliable VLC and V-VLC reception because the estimated channel affects equalization, detection, and subsequent receiver processing. Classical pilot-aided estimators, such as least squares (LS), are attractive because of their low complexity, while linear minimum mean square error (LMMSE) estimators can exploit second-order channel statistics to reduce noise sensitivity [11]. Nevertheless, practical VLC observations are not always well described by signal-independent additive white Gaussian noise. In intensity-modulation/direct-detection links, signal-dependent shot noise (SDSN) arises from photon detection randomness and depends on the received optical signal. This dependence can degrade conventional estimators when it is not explicitly considered [12]–[14].

Recent VLC channel-estimation works have considered optical noise mechanisms and learning-aided estimation. LS, ML, MAP, LMMSE, and MMSE estimators have been studied under input- or signal-dependent noise, and analytical bounds have been used to quantify the estimation error [12], [13]. Machine-learning-aided estimators have also been proposed

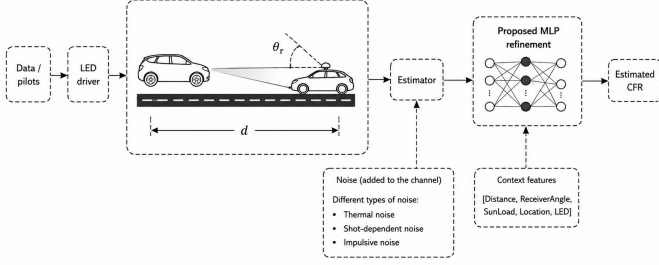


Fig. 1. System model of the considered V-VLC CFR observation framework.

to improve VLC channel estimation under SDSN [15]. In parallel, neural and deep-learning methods have been investigated for VLC channel estimation, denoising, equalization, demodulation, modulation classification, and symbol detection [16]–[20]. However, most of these works focus on indoor VLC links, scalar channel-gain estimation, or simulated channel settings. Pilot-aided reconstruction of measured V-VLC CFR vectors under composite optical noise remains largely unexplored. Impulsive disturbances introduce an additional source of mismatch. Sporadic high-amplitude perturbations are commonly modeled using Bernoulli–Gaussian processes [21]. Under such disturbances, pilot averaging reduces part of the noise but may also hide unstable pilot samples. Hence, pilot-block features, such as dispersion, median level, and range across repeated pilots, can provide information that is not retained by the averaged LS estimate alone. Moreover, V-VLC contextual descriptors remain useful because the measured CFR is strongly linked to distance, receiver angle, ambient illumination, lane condition, and LED configuration. This motivates a hybrid estimator that combines statistical filtering, pilot-block information, and physical context.

Motivated by these observations, this paper investigates pilot-aided reconstruction of measured V-VLC CFR vectors under composite optical noise. The measured CFR data reported in [8] are used as clean channel references, and noisy pilot observations are generated under thermal noise, SDSN, and Bernoulli–Gaussian impulsive noise. Instead of replacing classical estimation with a fully data-driven predictor, the proposed approach first computes an empirical LMMSE estimate from the noisy pilots. A residual multilayer perceptron then corrects the remaining error in the dB domain using the LMMSE estimate, pilot-block features, V-VLC contextual descriptors, and the operating SNR. The resulting estimator is referred to as Neural-Residual LMMSE (NR-LMMSE). The main contributions of this paper are summarized as follows:

- A pilot-aided V-VLC CFR reconstruction problem is formulated using measured CFR data over 19 frequency samples, rather than a scalar channel-gain target.
- A composite observation model is considered, including thermal noise, signal-dependent shot noise, and Bernoulli–Gaussian impulsive noise.
- A hybrid NR-LMMSE estimator is proposed, where an

empirical LMMSE estimate is refined by a residual MLP using pilot-block features, V-VLC context, and SNR information.

- The proposed estimator is evaluated against LS, empirical LMMSE, bias-corrected LMMSE, LS-based neural residual correction, and context-only learning baselines under multiple noise scenarios.
- The results show that NR-LMMSE provides its main benefit under composite signal-dependent and impulsive noise, reducing both dB-domain reconstruction error and the high-error tail of the sample-wise NMSE distribution.

II. SYSTEM AND COMPOSITE NOISE MODEL

Fig. 1 illustrates the considered pilot-aided V-VLC CFR observation model. A known pilot signal is emitted by the LED/headlamp source and received by an optical receiver mounted on the receiving vehicle. The measured CFR vector is used as the reference channel response, while the corresponding pilot observations are corrupted by composite optical noise.

A. Measured V-VLC CFR Representation

We consider a measured V-VLC link characterized by CFR magnitudes over $K = 19$ frequency samples from 200 kHz to 2 MHz with 100 kHz spacing. For the m -th measured condition, $m = 1, \dots, M$, the linear CFR magnitude vector is

$$\mathbf{h}_m = [h_m(f_1), h_m(f_2), \dots, h_m(f_K)]^T \in \mathbb{R}_+^K, \quad (1)$$

where $f_1 = 200$ kHz, $f_K = 2$ MHz, and $h_m(f_k)$ denotes the CFR magnitude at frequency sample f_k . The measured CFR entries are stored in dB and converted to linear magnitude as $h_m(f_k) = 10^{h_m^{\text{dB}}(f_k)/20}$. Each CFR vector is associated with the contextual descriptor

$$\mathbf{c}_m = [d_m, \theta_m, s_m, \ell_m, e_m]^T, \quad (2)$$

where d_m , θ_m , s_m , ℓ_m , and e_m denote the inter-vehicle distance, receiver inclination angle, sun-load sensor value, occupied-lane condition, and LED identifier, respectively. The measured dataset is denoted by $\mathcal{D} = \{(\mathbf{h}_m, \mathbf{c}_m)\}_{m=1}^M$.

B. Pilot Observation Model

For each measured CFR vector, known pilots are observed over the K frequency samples. The scalar observation at frequency f_k for the q -th pilot/noise realization is modeled as

$$y_{m,k,q} = x_k h_m(f_k) + n_{m,k,q}, \quad (3)$$

where m , k , and q index the measured condition, frequency sample, and pilot/noise realization, respectively. The variable x_k is the known pilot symbol and $n_{m,k,q}$ is the composite noise sample. For compact notation, let $\mathbf{y}_{m,q} = [y_{m,1,q}, \dots, y_{m,K,q}]^T$, $\mathbf{x} = [x_1, \dots, x_K]^T$, and $\mathbf{n}_{m,q} = [n_{m,1,q}, \dots, n_{m,K,q}]^T$. Then,

$$\mathbf{y}_{m,q} = \mathbf{x} \odot \mathbf{h}_m + \mathbf{n}_{m,q}, \quad (4)$$

where $\odot$ denotes element-wise multiplication. The complete pilot observation block is

$$\mathbf{Y}_m = [\mathbf{y}_{m,1}, \mathbf{y}_{m,2}, \dots, \mathbf{y}_{m,N_p}] \in \mathbb{R}^{K \times N_p}. \quad (5)$$

In the numerical experiments, unit-amplitude pilots are used, i.e., $x_k = 1$ for all k .

C. Composite Noise Model

The composite noise in (3) is modeled as

$$n_{m,k,q} = n_{m,k,q}^{\text{th}} + n_{m,k,q}^{\text{sdsn}} + n_{m,k,q}^{\text{imp}}, \quad (6)$$

where the three terms denote thermal noise, signal-dependent shot noise (SDSN), and impulsive noise, respectively. For the m -th measured condition, the average received pilot power is $P_m = K^{-1} \sum_{k=1}^K |x_k h_m(f_k)|^2$. The thermal component is modeled as $n_{m,k,q}^{\text{th}} \sim \mathcal{N}(0, \sigma_{\text{th},m}^2)$, with $\sigma_{\text{th},m}^2 = P_m/\gamma$ and $\gamma = 10^{\text{SNR}/10}$. The SDSN component is modeled as

$$n_{m,k,q}^{\text{sdsn}} = \sqrt{|x_k h_m(f_k)|} \epsilon_{m,k,q}, \quad \epsilon_{m,k,q} \sim \mathcal{N}(0, \zeta^2 \sigma_{\text{sdsn},m}^2), \quad (7)$$

where ζ^2 is the SDSN scaling factor. The reference SDSN variance is $\sigma_{\text{sdsn},m}^2 = P_m/\gamma_{\text{ref}}$, with $\gamma_{\text{ref}} = 10^{25/10}$. This reference level is kept independent of the target thermal SNR, so increasing the thermal SNR does not remove the signal-dependent noise contribution. The value $\gamma_{\text{ref}} = 10^{25/10}$ is used as a representative reference operating point for the signal-dependent component. Finally, the impulsive component follows a Bernoulli–Gaussian model,

$$n_{m,k,q}^{\text{imp}} = b_{m,k,q} g_{m,k,q}, \quad b_{m,k,q} \sim \text{Bernoulli}(p_{\text{imp}}), \quad (8)$$

where $g_{m,k,q} \sim \mathcal{N}(0, \rho_I |x_k h_m(f_k)|^2)$. The parameter p_{imp} is the impulse occurrence probability, and ρ_I controls the relative impulsive-noise power.

III. BASELINE AND PROPOSED ESTIMATORS

This section defines the estimators evaluated in this work. LS and empirical LMMSE are used as pilot-aided baselines. A bias-corrected LMMSE baseline is included to isolate static dB-domain offset correction. The proposed estimator is the Neural-Residual LMMSE (NR-LMMSE), which refines the empirical LMMSE estimate using pilot-block features, V-VLC context, and SNR information.

A. LS and Empirical LMMSE Estimation

From the observation model in (3), the pilot-normalized vector associated with the q -th realization is

$$\mathbf{z}_{m,q} = \begin{bmatrix} \frac{y_{m,1,q}}{x_1}, \dots, \frac{y_{m,K,q}}{x_K} \end{bmatrix}^T \in \mathbb{R}^K. \quad (9)$$

The LS estimate is obtained by averaging the normalized pilot vectors:

$$\hat{\mathbf{h}}_m^{\text{LS}} = \frac{1}{N_p} \sum_{q=1}^{N_p} \mathbf{z}_{m,q}. \quad (10)$$

LS removes the pilot scaling and reduces random perturbations by averaging, but it does not exploit cross-frequency CFR correlation.

The empirical LMMSE estimator uses $\mathbf{v}_m = \hat{\mathbf{h}}_m^{\text{LS}}$ as its observation. Let $\mathcal{I}_{\text{tr}}$ denote the training index set. The empirical means $\boldsymbol{\mu}_h$ and $\boldsymbol{\mu}_v$, and the covariance matrices $\mathbf{C}_{hv}$ and $\mathbf{C}_{vv}$, are estimated from $\{(\mathbf{h}_m, \mathbf{v}_m)\}_{m \in \mathcal{I}_{\text{tr}}}$. The LMMSE estimate is

$$\hat{\mathbf{h}}_m^{\text{LMMSE}} = \boldsymbol{\mu}_h + \mathbf{C}_{hv} (\mathbf{C}_{vv} + \lambda \mathbf{I}_K)^{-1} (\mathbf{v}_m - \boldsymbol{\mu}_v), \quad (11)$$

where $\lambda > 0$ is a regularization coefficient and $\mathbf{I}_K$ is the $K \times K$ identity matrix. The term empirical indicates that the second-order statistics are learned from the measured training data.

B. Bias-Corrected LMMSE Baseline

A bias-corrected LMMSE baseline is used to check whether the LMMSE residual error can be reduced by a static per-frequency correction in the dB domain. The validation bias is

$$\mathbf{b}_{\text{val}} = \frac{1}{|\mathcal{I}_{\text{val}}|} \sum_{m \in \mathcal{I}_{\text{val}}} (\mathbf{h}_m^{\text{dB}} - \hat{\mathbf{h}}_m^{\text{LMMSE,dB}}), \quad (12)$$

where $\mathcal{I}_{\text{val}}$ is the validation index set and $\hat{\mathbf{h}}_m^{\text{LMMSE,dB}}$ is the dB-domain form of $\hat{\mathbf{h}}_m^{\text{LMMSE}}$. The corrected estimate is

$$\hat{\mathbf{h}}_m^{\text{BC,dB}} = \hat{\mathbf{h}}_m^{\text{LMMSE,dB}} + \mathbf{b}_{\text{val}}. \quad (13)$$

This baseline separates validation-calibrated bias correction from the nonlinear residual correction used in NR-LMMSE.

C. Pilot-Block Features

The pilot observation block $\mathbf{Y}_m$ in (5) contains N_p noisy realizations for each measured CFR vector. Before averaging, we form the normalized pilot block $\mathbf{Z}_m = [\mathbf{z}_{m,1}, \dots, \mathbf{z}_{m,N_p}] \in \mathbb{R}^{K \times N_p}$. To retain pilot-level variability, three per-frequency statistics are computed across the N_p realizations: the standard deviation $\sigma_{m,k}^{\text{p}} = \text{std}_q(z_{m,k,q})$, the median $\nu_{m,k}^{\text{p}} = \text{median}_q(z_{m,k,q})$, and the range $r_{m,k}^{\text{p}} = \max_q(z_{m,k,q}) - \min_q(z_{m,k,q})$. The pilot-feature vector is then defined as

$$\mathbf{s}_m^{\text{p}} = [\boldsymbol{\sigma}_m^{\text{p}}, \boldsymbol{\nu}_m^{\text{p}}, \mathbf{r}_m^{\text{p}}] \in \mathbb{R}^{3K}, \quad (14)$$

where $\boldsymbol{\sigma}_m^{\text{p}}$, $\boldsymbol{\nu}_m^{\text{p}}$, and $\mathbf{r}_m^{\text{p}}$ collect the corresponding statistics over the K frequency samples. These features are used only by the neural residual stage.

D. Proposed NR-LMMSE Refinement

The proposed NR-LMMSE estimator refines the empirical LMMSE estimate in the dB domain. The LMMSE estimate is first converted as

$$\hat{\mathbf{h}}_m^{\text{LMMSE,dB}} = 20 \log_{10} \left(\max \left(\hat{\mathbf{h}}_m^{\text{LMMSE}}, \epsilon_0 \right) \right), \quad (15)$$

where the maximum is applied element-wise and $\epsilon_0 = 10^{-12}$ avoids numerical singularities. The residual target is

$$\mathbf{r}_m^{\text{dB}} = \mathbf{h}_m^{\text{dB}} - \hat{\mathbf{h}}_m^{\text{LMMSE,dB}}. \quad (16)$$

Let $\Delta_{\Theta}(\cdot)$ denote the residual MLP with trainable parameters Θ . The predicted residual is defined as

$$\hat{\mathbf{r}}_m^{\text{dB}} = \Delta_{\Theta} \left(\hat{\mathbf{h}}_m^{\text{LMMSE,dB}}, \mathbf{s}_m^{\text{p}}, \mathbf{c}_m, \text{SNR} \right). \quad (17)$$

The NR-LMMSE estimate is then given by

$$\hat{\mathbf{h}}_m^{\text{NR,dB}} = \hat{\mathbf{h}}_m^{\text{LMMSE,dB}} + \hat{\mathbf{r}}_m^{\text{dB}}. \quad (18)$$

Thus, the neural network learns only the residual correction left after empirical LMMSE filtering.

The network is trained by minimizing the smooth- ℓ_1 loss

$$\Theta^* = \arg \min_{\Theta} \sum_{m \in \mathcal{I}_{\text{tr}}} \mathcal{L}_{\text{SL1}}(\mathbf{r}_m^{\text{dB}} - \hat{\mathbf{r}}_m^{\text{dB}}). \quad (19)$$

At test time, the measured CFR vector is not used by the estimator. It is used only as a reference for performance evaluation.

IV. EXPERIMENTAL SETUP

This section describes the experimental protocol used to evaluate the estimators defined in Section III. The measured CFR representation, pilot observation model, and composite noise model follow Section II. Here, we specify the dataset settings, simulation parameters, compared methods, learning configuration, implementation details, and evaluation metrics. The experiments use the measured V-VLC CFR dataset reported in [8], which was collected through practical road measurements and contains $M = 1345$ channel conditions. Each measured condition includes a CFR magnitude vector over $K = 19$ frequency samples from 200 kHz to 2 MHz with 100 kHz spacing. The associated contextual descriptor includes the inter-vehicle distance, receiver inclination angle, sun-load sensor value, occupied-lane condition, and LED identifier, as defined in (2). In this work, the dataset is used as a set of clean reference CFR vectors for the pilot-aided reconstruction problem. This differs from the data-driven channel-modeling objective of [8], where the CFR is inferred mainly from physical and environmental descriptors.

Noisy pilot observations are generated from (3) using $N_p = 8$ pilot/noise realizations per measured channel condition. The dataset is randomly divided into training, validation, and test sets using a 70/15/15 split. All reported results are averaged over ten evaluation seeds, indexed from 0 to 9. The SNR ranges from 0 to 30 dB in 5 dB steps. Unless otherwise stated, the main composite-noise setting uses $\zeta^2 = 1$, $\gamma_{\text{ref}} = 10^{25/10}$, $p_{\text{imp}} = 0.03$, and $\rho_I = 10$. The value $\rho_I = 10$ represents rare but high-amplitude impulsive perturbations, where each impulse has a variance ten times the local received signal power. The evaluated estimators are LS, empirical LMMSE, bias-corrected LMMSE (BC-LMMSE), LS-based neural residual correction (LS-NR), context-only neural prediction (Ctx-only), and the proposed NR-LMMSE estimator. The LS estimator performs pilot-normalized least-squares averaging. The LMMSE estimator uses training-set second-order statistics. BC-LMMSE applies validation-set dB-domain offset correction to the LMMSE output. LS-NR applies neural residual correction directly to the LS estimate. Ctx-only uses only the V-VLC contextual information and SNR as neural-network inputs. NR-LMMSE refines the empirical LMMSE estimate using a neural residual correction. The neural models use the same training protocol for a fair comparison. All inputs

are standardized using training-set statistics only, and the validation set is used for early stopping and for the bias correction in BC-LMMSE.

The residual MLP is trained in the dB domain using the smooth- ℓ_1 loss in (19). For NR-LMMSE, the input consists of the empirical LMMSE estimate, pilot-block features, contextual descriptors, and the SNR value. The output dimension is $K = 19$, corresponding to the residual correction over all CFR frequency samples. For all neural estimators, the SNR value in dB is normalized by 30 dB and used as an input feature. The neural models are trained with AdamW using a maximum of 80 epochs, a batch size of 128, dropout of 0.10, and early stopping with patience 12. For each SNR value, independent noisy pilot blocks are generated from the measured CFR vectors. Each block contains $N_p = 8$ pilot/noise realizations, as defined in (5). To augment the evaluation protocol, 7, 2, and 30 independent noisy pilot blocks are generated per measured condition for the training, validation, and test sets, respectively. This provides multiple noisy observations of the same measured CFR vector while keeping the underlying reference CFR fixed. Numerical context features are standardized using training-set statistics, while categorical descriptors are encoded using one-hot encoding. The same preprocessing fitted on the training set is applied to the validation and test sets. Extra dataset columns that directly summarize the CFR are not used as inputs, in order to avoid information leakage. For the neural residual estimators, both input features and residual targets are standardized using training-set statistics only. The clipping constant used before dB conversion is set to $\epsilon_0 = 10^{-12}$.

Performance is evaluated using global NMSE, dB-domain MAE, and the 95th percentile of sample-wise NMSE. For an evaluation set $\mathcal{I} * \text{ev}$, the global NMSE is

$$\text{NMSE} = \frac{\sum_{*m \in \mathcal{I}_{\text{ev}}} \|\mathbf{h} * m - \hat{\mathbf{h}} * m\|_2^2}{\sum_{*m \in \mathcal{I} * \text{ev}} \|\mathbf{h}_m\|_2^2}. \quad (20)$$

The MAE is computed between $\mathbf{h}_m^{\text{dB}}$ and $\hat{\mathbf{h}}_m^{\text{dB}}$. The 95th percentile of sample-wise NMSE is used to quantify the high-error tail.

V. RESULTS AND DISCUSSION

A. Composite-Noise Performance

Fig. 2 reports the NMSE versus SNR under the composite noise model, including thermal noise, SDSN, and Bernoulli–Gaussian impulsive noise. LS remains the least accurate method because pilot averaging alone cannot fully suppress signal-dependent and impulsive perturbations. Empirical LMMSE improves the estimate by exploiting cross-frequency statistics learned from the measured training data. NR-LMMSE further reduces the residual error, especially at moderate and high SNR values.

Table I summarizes the composite-noise performance. Compared with empirical LMMSE, NR-LMMSE reduces the global NMSE from 3.74×10^{-3} to 2.46×10^{-3} , corresponding to a 34.2% reduction. The MAE decreases from 0.858 dB to 0.296 dB, and the P95 sample-wise NMSE decreases from

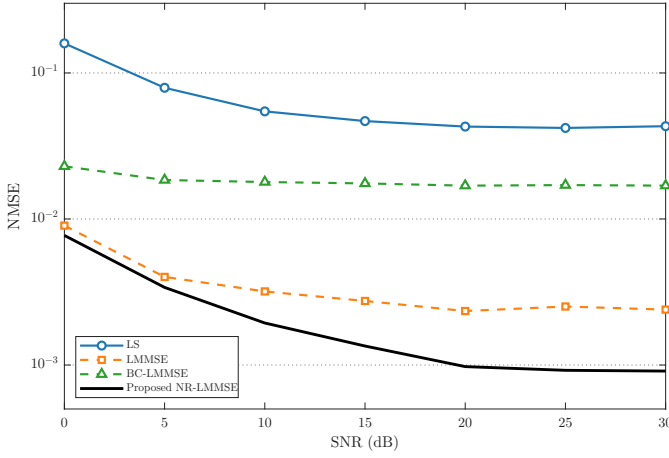


Fig. 2. NMSE versus SNR under composite thermal, SDSN, and Bernoulli-Gaussian impulsive noise. Curves report the mean over ten evaluation seeds.

TABLE I

MAIN PERFORMANCE UNDER COMPOSITE NOISE. VALUES ARE AVERAGED OVER TEN EVALUATION SEEDS AND OVER THE SNR GRID FROM 0 TO 30 dB.

Method	NMSE	MAE (dB)	P95 NMSE
LS	6.69×10^{-2}	2.213	1.78×10^{-1}
LMMSE	3.74×10^{-3}	0.858	1.59×10^{-1}
BC-LMMSE	1.83×10^{-2}	0.859	9.85×10^{-2}
NR-LMMSE	2.46×10^{-3}	0.296	1.11×10^{-2}

1.59×10^{-1} to 1.11×10^{-2} . BC-LMMSE reduces the P95 NMSE compared with LMMSE, but its global NMSE increases markedly. This behavior is expected: the offset in (13) is calibrated in the dB domain, and after conversion to the linear domain the correction amplifies errors at frequency samples with large CFR magnitude, which dominate the linear NMSE in (20). A static per-frequency dB offset is therefore insufficient, and can even be counterproductive, under composite noise. This further motivates the nonlinear, observation-dependent residual correction used in NR-LMMSE.

B. Impact of the Noise Model

Fig. 3 compares empirical LMMSE and NR-LMMSE under AWGN, AWGN+SDSN, and composite noise. Under AWGN and AWGN+SDSN, the two methods remain close because empirical LMMSE is already effective under these less impulsive observation models. Averaged over SNR values and ten evaluation seeds, the NMSE reduction of NR-LMMSE relative to empirical LMMSE is about 4.6% under AWGN and 4.1% under AWGN+SDSN. Under composite noise, the reduction increases to about 34.5%, showing that the residual correction is mainly beneficial when impulsive perturbations are present.

C. Sample-Wise Error Distribution

Fig. 4 shows the CDF of sample-wise NMSE at 15 dB under composite noise. NR-LMMSE shifts the distribution toward lower error values compared with LS and empirical LMMSE. This behavior is consistent with the P95 NMSE reduction

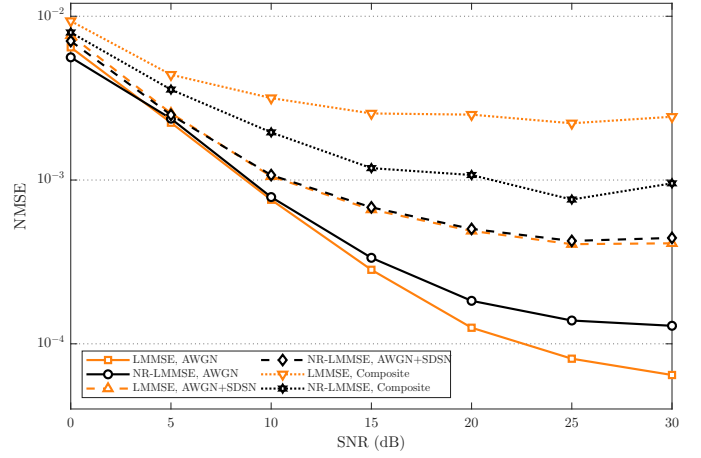


Fig. 3. Noise-model comparison between empirical LMMSE and NR-LMMSE under AWGN, AWGN+SDSN, and composite thermal, SDSN, and impulsive noise. Curves report the mean over ten evaluation seeds.

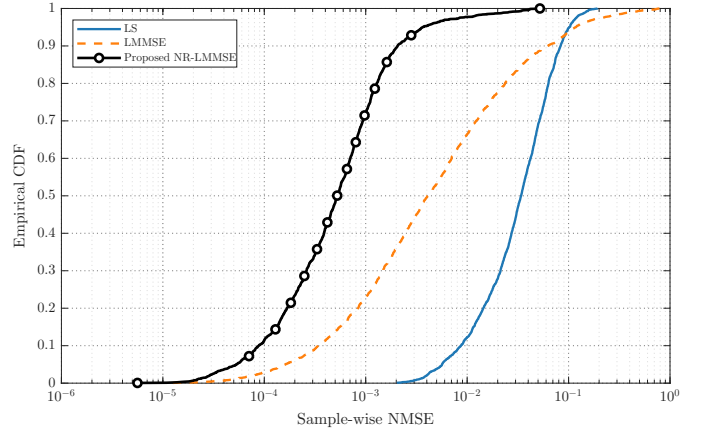


Fig. 4. CDF of sample-wise NMSE under composite noise at 15 dB, pooled over ten evaluation seeds.

reported in Table I, indicating that the gain is not limited to the average NMSE.

D. Ablation Study and Complexity

Table II reports the ablation study at 15 dB under composite noise. The context-only model gives the largest error, indicating that contextual descriptors alone are insufficient for pilot-aided CFR reconstruction. LS-NR improves the dB-domain error compared with LS, but its NMSE remains higher than that of the LMMSE-based residual formulation. BC-LMMSE reduces part of the LMMSE bias, but it does not sufficiently control the high-error tail. NR-LMMSE gives the lowest NMSE, MAE, and P95 NMSE among the evaluated methods.

At 15 dB, NR-LMMSE reduces the NMSE by 50.9%, the MAE by 72.1%, and the P95 NMSE by 97.4% relative to empirical LMMSE. NR-LMMSE uses approximately 50.5k trainable parameters and requires 0.0198 ± 0.0038 ms per sample. This runtime remains close to that of LS-NR and

TABLE II
ABLATION UNDER COMPOSITE NOISE AT 15 dB. VALUES ARE AVERAGED
OVER TEN EVALUATION SEEDS.

Method	NMSE	MAE (dB)	P95 NMSE
LS	4.68×10^{-2}	1.531	1.02×10^{-1}
LMMSE	2.75×10^{-3}	0.813	1.54×10^{-1}
BC-LMMSE	1.75×10^{-2}	0.819	9.36×10^{-2}
LS-NR	1.62×10^{-2}	0.652	4.67×10^{-2}
Ctx-only	4.62×10^{-1}	3.594	14.2
NR-LMMSE	1.35×10^{-3}	0.227	4.04×10^{-3}

adds only a small neural correction overhead to the empirical LMMSE stage.

VI. CONCLUSION

This paper proposed NR-LMMSE for pilot-aided reconstruction of measured V-VLC CFR vectors under composite optical noise. The method refines an empirical LMMSE estimate through a dB-domain neural residual correction using pilot-block features, V-VLC context, and SNR information.

Results show that NR-LMMSE provides limited additional gain under AWGN and AWGN+SDSN, where empirical LMMSE is already effective. The main benefit appears under composite thermal, signal-dependent, and impulsive noise, where the residual correction improves both average reconstruction accuracy and high-error tail robustness relative to empirical LMMSE. Since the present evaluation relies on a random split of a single measurement campaign, future work will consider physically disjoint splits, such as unseen LED configurations or distance ranges, additional measured V-VLC scenarios, and mobility-aware CFR tracking.

REFERENCES

- [1] H. Haas, L. Yin, Y. Wang, and C. Chen, "What is LiFi?," *Journal of Lightwave Technology*, vol. 34, no. 6, pp. 1533–1544, Mar. 2016.
- [2] L. E. M. Matheus, A. B. Vieira, L. F. M. Vieira, M. A. M. Vieira, and O. Gnawali, "Visible light communication: Concepts, applications and challenges," *IEEE Communications Surveys & Tutorials*, vol. 21, no. 4, pp. 3204–3237, Fourthquarter 2019.
- [3] IEEE Standard for Local and Metropolitan Area Networks—Part 15.7: Short-Range Wireless Optical Communication Using Visible Light, IEEE Std 802.15.7-2011, Sept. 2011.
- [4] A. Memedi and F. Dressler, "Vehicular visible light communications: A survey," *IEEE Communications Surveys & Tutorials*, vol. 23, no. 1, pp. 161–181, Firstquarter 2021.
- [5] M. Uysal, Z. Ghassemloooy, A. Bekkali, A. Kadri, and H. Menouar, "Visible light communication for vehicular networking: Performance study of a V2V system using a measured headlamp beam pattern model," *IEEE Vehicular Technology Magazine*, vol. 10, no. 4, pp. 45–53, Dec. 2015.
- [6] M. Elamassie, M. Karbalayghareh, F. Miramirkhani, R. C. Kizilirmak, and M. Uysal, "Effect of fog and rain on the performance of vehicular visible light communications," in *Proc. IEEE 87th Vehicular Technology Conference (VTC Spring)*, Porto, Portugal, Jun. 2018, pp. 1–6.
- [7] A. H. Wheeb, S. Maurya, S. Karimullah, F. Shaik, S. Chowdhury, B. V. V. Siva Prasad, A. Sinha, A. Sood, and S. A. Hussain, "Improved spectral efficiency and channel estimation parameters in visible light vehicular communication by integrating simulation of urban mobility data," *IEEE Access*, vol. 13, pp. 70828–70848, 2025.
- [8] B. Turan and S. Coleri, "Machine learning based channel modeling for vehicular visible light communication," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 10, pp. 9659–9672, Oct. 2021.
- [9] F. M. Alsalamy, Z. Ahmad, S. Zvanovec, P. A. Haigh, O. C. Haas, and S. Rajbhandari, "Statistical channel modelling of dynamic vehicular visible light communication system," *Vehicular Communications*, vol. 29, p. 100339, 2021.
- [10] M. S. Amjad, C. Tebruegge, A. Memedi, S. Kruse, C. Kress, J. C. Scheytt, and F. Dressler, "Towards an IEEE 802.11 compliant system for outdoor vehicular visible light communications," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 6, pp. 5749–5761, Jun. 2021.
- [11] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1993.
- [12] M. Yaseen, M. Alsmadi, A. E. Canbilen, and S. Ikki, "Visible light communication with input-dependent noise: Channel estimation, optimal receiver design and performance analysis," *Journal of Lightwave Technology*, vol. 39, no. 23, pp. 7406–7416, Dec. 2021.
- [13] M. Yaseen, A. E. Canbilen, and S. Ikki, "Channel estimation in visible light communication systems: The effect of input signal-dependent noise," *IEEE Transactions on Vehicular Technology*, vol. 72, no. 11, pp. 14330–14340, Nov. 2023.
- [14] M. Yaseen, M. Elamassie, S. Ikki, and M. Uysal, "Signal-dependent shot and relative intensity noise in channel estimation of laser diode-based indoor VLC systems," *IEEE Transactions on Communications*, vol. 73, no. 1, pp. 498–509, Jan. 2025.
- [15] S. H. ElFar, M. Yaseen, and S. Ikki, "A novel machine learning algorithm with mathematical modeling for channel estimation in VLC systems," *IEEE Wireless Communications Letters*, vol. 14, no. 7, pp. 2084–2088, Jul. 2025.
- [16] A. Yesilkaya, O. Karatalay, A. S. Ogrenci, and E. Panayirci, "Channel estimation for visible light communications using neural networks," *arXiv preprint arXiv:1805.08060*, 2018.
- [17] S. Ma, J. Dai, S. Lu, H. Li, H. Zhang, C. Du, and S. Li, "Signal demodulation with machine learning methods for physical layer visible light communications: Prototype platform, open dataset, and algorithms," *IEEE Access*, vol. 7, pp. 30588–30598, 2019.
- [18] A. Al-Imran, M. Z. Chowdhury, M. I. Joha, M. M. Rahman, M. S. Kabir, and Y. M. Jang, "Machine learning and deep learning in VLC systems: A comprehensive survey," *IEEE Open Journal of the Communications Society*, vol. 6, pp. 7955–8006, 2025.
- [19] M. H. Rahman, M. Z. Chowdhury, I. B. K. Y. Utama, and Y. M. Jang, "Channel estimation for indoor massive MIMO visible light communication with deep residual convolutional blind denoising network," *IEEE Transactions on Cognitive Communications and Networking*, vol. 9, no. 3, pp. 683–694, Jun. 2023.
- [20] H. Zhou, W. Zhang, Z. Tong, and H. Wang, "Channel estimation method for VLC systems based on residual network and Kalman filtering," in *Proc. 2nd International Conference on Industrial Automation and Robotics (IAR)*, Tianjin, China, Dec. 2025, pp. 1–6.
- [21] B. Han and H. D. Schotten, "A fast blind impulse detector for Bernoulli-Gaussian noise in underspread channel," in *Proc. IEEE International Conference on Communications (ICC)*, Kansas City, MO, USA, May 2018, pp. 1–6.